Exchange Rates and Fundamentals: A General Equilibrium Exploration

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Introduction

- A naive random-walk (RW) statistical model of nominal exchange rates has beaten equilibrium open-economy macro models for last 30 years.


- Are equilibrium economic models useless for understanding exchange rate movements?

- But then what can we say about impacts of macro policies on foreign exchange markets without an equilibrium model?
Motivation

- Can we theoretically establish RW exchange rates in a general equilibrium model?

- Can such a general equilibrium model empirically explain not only RW exchange rates but also economic fundamentals jointly?

- What structural shocks would be the main drivers of RW exchange rates?
Past studies: RW EX in a partial equilibrium model

- Engel and West (2005, JPE)

- A classical partial equilibrium asset approach can generate a near RW behavior of exchange rates under the following two conditions
  1. Economic fundamentals are I(1).
  2. Discount factor approaches one.
Past studies: RW EX in a micro-founded model

  - A two-country incomplete-market model.
  - Money demand function from money-in-utility.
  - The present value model (PVM) of exchange rates

\[
\ln S_t = (1 - \kappa) \sum_{i=0}^{\infty} \kappa^i E_t (\ln M_{t+i} - \ln C_{t+i})
\]

- Fundamental \( \ln M_t - \ln C_t \) needs to be I(1).
Past studies: RW EX in a micro-founded model (cont’d)

- Because the fundamentals are I(1), they have the Beveridge-Nelson permanent and transitory decomposition.

- The BN permanent component is RW.

- When the discount factor is close to 1, the BN trend component dominates the present value calculation by the market participants.
Two arguments

1. Unit discount factor is sufficient but not necessary for a RW exchange rate.
   - If economic fundamentals follow RWs, so does an exchange rate, regardless of the size of the discount factor.
Two arguments (cont’d)

2 Implications of unit discount factor for endogenous economics fundamentals?

- EW, NR, and BMW investigate partial equilibrium models.
- Cross-country differentials in consumption and nominal interest rate are treated as exogenous economic fundamentals.
- But they are endogenous fundamentals in general equilibrium.
- What does the unit discount factor imply for endogenous economic fundamentals?
A canonical two-country DSGE model

- Investigating a canonical two-country DSGE model encompassing EW, NR, and BMW as much as possible.
  - Money-in-utility for money demand function
  - Permanent money supply shocks
  - Incomplete markets with debt elastic risk premium
  - Cointegrated productivity shocks
  - Exogenous PPP deviations, i.e., real exchange rate shocks.
Theoretical implications

- The model theoretically preserves equilibrium RW property of exchange rates when fundamentals are $I(1)$ and the discount factor is close to 1.

  - However, if exogenous economic fundamentals follow RWs, so does an exchange rate, regardless of the size of the discount factor.

- Need to estimate the discount factor and the degrees of persistence of exogenous economic fundamentals jointly to evaluate the fit of the model to RW exchange rate.

  - Some economic fundamentals are unobservable.

  - Crucial role of persistent unobservable monetary shocks: EW, BMW, Sarno and Sojli (2009, JMCB)
Theoretical implications (cont’d)

At the limit of the unit discount factor, the model implies three unrealistic restrictions:

1. The unique fundamental driver of RW exchange rates is permanent shocks to money supply differential.

2. Backus and Smith’s puzzle: relative consumption is perfectly correlated with real exchange rate.

3. Counterfactually large volatilities of monetary disturbances due to perfectly flat money demand function, i.e., the liquidity trap at the steady state.
Empirical results

- These restrictions matter empirically:
  - Bayesian posterior inferences of the DSGE model using post-Bretton Woods sample of Canada and the U.S.
  - The discount factor is estimated around 0.54. Much smaller than the past estimates and empirically implausible.
  - Money demand shock is estimated to be almost RW and the main driver of exchange rate

- Hence, there is difficulty to explain RW exchange rates and economic fundamentals jointly and consistently within a canonical two-country DSGE model.
Key ingredients of the model: Households

Households in country $i$ maximize the lifetime MIU

$$\sum_{j=0}^{\infty} \beta^j E_t \left\{ \ln C_{i,t+j} + \phi_{i,t} \ln \left( \frac{M_{i,t+j}}{P_{i,t+j}} \right) \right\}, \quad 0 < \beta < 1, \quad \text{for } i = h, f$$

where $\phi_t$ is the money demand shock

$$\ln \phi_{i,t} = (1 - \rho_\phi) \ln \phi + \rho_\phi \ln \phi_{i,t-1} + \epsilon_{\phi,t}^i$$
Key ingredients of the model: Households

- Home country’s budget constraint

\[ B_{h,t} + S_t B_{h,t}^f + P_{h,t} C_{h,t} + M_{h,t} = (1 + r_{h,t-1}^h) B_{h,t-1}^h + S_t (1 + r_{h,t-1}^f) B_{h,t-1}^f + M_{h,t-1} + P_{h,t} Y_{h,t} + T_{h,t}. \]

or the foreign country’s budget constraint

\[ \frac{B_{f,t}^h}{S_t} + B_{f,t}^f + P_{f,t} C_{f,t} + M_{f,t} = (1 + r_{f,t-1}^h) \frac{B_{f,t-1}^h}{S_t} + (1 + r_{f,t-1}^f) B_{f,t-1}^f + M_{f,t-1} + P_{f,t} Y_{f,t} + T_{f,t}, \]
Key ingredients of the model: Monetary policy

- Monetary policy

\[ M_{i,t} = m_{i,t} M_{i,t}^\tau, \quad \text{for } i = h, f, \]

where \( M_{i,t}^\tau \) is the permanent component and \( m_{i,t} \) is the transitory component:

\[ \Delta \ln M_{i,t}^\tau = \ln \gamma_M + \rho_M \Delta \ln M_{i,t-1}^\tau + \epsilon_{M,t}^i, \]

\[ \ln m_{i,t} = \epsilon_{m,t}^i. \]
Key ingredients of the model: Endowment

- Endowment

\[ Y_{i,t} = y_{i,t}A_{i,t} \]

where \( A_{i,t} \) is the permanent component and \( y_{i,t} \) is the transitory component:

\[ \ln y_{i,t} = \ln y_i + \epsilon_{y,t}^i. \]
Key ingredients of the model: Endowment (cont’d)

- The two countries’ permanent TFP are cointegrated with the ECMs

\[
\Delta \ln A_{h,t} = \ln \gamma_A - \frac{\lambda}{2} (\ln A_{h,t-1} - \ln A_{f,t-1}) + \epsilon^h_{A,t},
\]

\[
\Delta \ln A_{f,t} = \ln \gamma_A + \frac{\lambda}{2} (\ln A_{h,t-1} - \ln A_{f,t-1}) + \epsilon^f_{A,t}.
\]

- Cross-country TFP differential \( \ln a_t = \ln A_{h,t} - \ln A_{f,t} \) is I(0) but if the error correction speed \( \lambda \) is small, it is approximately I(1) as claimed by NR.

\[
\ln a_t = (1 - \lambda) \ln a_{t-1} + \epsilon^h_{A,t} - \epsilon^f_{A,t}
\]
Key ingredients of the model: PPP deviation shock

- PPP holds up to an exogenous PPP deviation shock

\[ S_t P_{f,t} = P_{h,t} q_t \]

where the real exchange rate fluctuates following the AR(1)

\[ \ln q_t = \rho_q \ln q_{t-1} + \epsilon_{q,t}. \]
Establishing RW-EX as a GE property

- Deriving an approximated analytical solution of the equilibrium currency return
  
  1. Stochastically detrended FONCs.
  2. Log-linearizing the detrended FONCs around the deterministic steady state.
  3. Solving the linear RE model to get the unique RE equilibrium path.
  4. Unwinding stochastic trend.

\[ \kappa = \frac{1}{1 + r^*} = \frac{\beta}{\gamma M} \]

- \( \kappa \) is the steady state market discount factor where \( r^* \) is the steady state nominal rate.
Establishing RW-EX as a GE property (cont’d)

- The equilibrium currency return is

\[
\Delta \ln S_t = \psi (1-\kappa) \tilde{b}_{t-1} + \frac{(1-\kappa)\rho_M}{1-\kappa\rho_M} \hat{\gamma}_{M,t-1} + \frac{(1-\kappa)(1-\rho_\phi)}{1-\kappa\rho_\phi} \ln \phi_{t-1}
- \frac{(1-\kappa)(1-\rho_m)}{1-\kappa\rho_m} \ln m_{t-1} + u_{s,t}.
\]

where \( u_{s,t} = \ln S_t - E_{t-1} \ln S_t \) is the RE error.

- When the discount factor \( \kappa \) approaches one, the currency return follows a Martingale difference

\[
\lim_{\kappa \to 1} E_{t-1} \Delta \ln S_t = 0
\]

- Exchange rate follows near RW at limit and the currency return lacks any dependence on past information.
Establishing RW-EX as a GE property (cont’d)

- When all monetary impulses follow RW, regardless of the size of $\kappa$, we get

$$\lim_{\rho_M \to 0, \rho_\phi, \rho_m \to 1} E_{t-1} \Delta \ln S_t = \psi(1 - \kappa)b_{t-1}$$

- Because parameter $\psi$ is usually calibrated by a quite small number, exchange rate is well approximated by a Martingale difference.

- Hence, the unit discount factor is sufficient but not necessary for RW exchange rate.
Structural interpretation of RW-EXs

- Reduced form rational expectations error $u_{s,t}$ is given as a linear combination of structural shocks.

- The rational expectations error is

\[
    u_{s,t} = \frac{1}{1 - \kappa \rho_M} \epsilon_{M,t} - \frac{1 - \beta \eta}{1 - \beta \eta (1 - \lambda)} \epsilon_{A,t} + \frac{1 - \kappa}{1 - \kappa \rho_m} \epsilon_{m,t} - \frac{1 - \beta \eta}{1 - \beta \eta \rho_y} \epsilon_{y,t} - \frac{1 - \kappa}{1 - \kappa \rho_q} \epsilon_{\phi,t} + \frac{1 - \beta \eta}{1 - \beta \eta \rho_q} \epsilon_{q,t}
\]

- Assume that $\beta \to 1$ makes $\kappa$ sufficiently close to 1.

- Then at limit

\[
    \lim_{\kappa \to 1} \Delta \ln S_t = \lim_{\kappa, \beta \to 1} u_{s,t} = \frac{1}{1 - \rho_M} \epsilon_{M,t}.
\]
Backus and Smith’s puzzle at the limit

The equilibrium dynamics of the consumption differential follow

\[
\Delta \ln C_t = \Delta \ln q_t - \psi(1 - \kappa)\tilde{b}_{t-1} + \frac{1 - \beta \eta}{1 - \beta \eta(1 - \lambda)} \epsilon_{A,t} \\
+ \frac{1 - \beta \eta}{1 - \beta \eta \rho_y} \epsilon_{y,t} + \frac{1 - \beta \eta}{1 - \beta \eta \rho_q} \epsilon_{q,t}.
\]

No monetary shocks matter for the consumption differential.

At the limit of the unit discount factor,

\[
\lim_{\kappa, \beta, \eta \to 1} \Delta \ln C_t = \Delta \ln q_t.
\]

At the limit, the consumption differential matches the PPP deviation, i.e., the real exchange rate shock (Backus and Smith’s puzzle: 1993 JIE)
Why the Backus and Smith puzzle here?
  - Each country’s consumption is determined by the PIH.
  - With the unit discount factor, only the permanent component of the endowment matters for consumption.
  - Any idiosyncratic shock to one country’s TFP is diffused to the other in the long run due to cointegration.
  - Two countries’ consumption respond to a TFP shock in the same way. No response of the consumption differential.
  - Only relative price, i.e., the RER, determines relative consumption.
The equilibrium dynamics of the interest rate follows

\[(1 + r_t) = (1 - \kappa) \left( \frac{\rho_M}{1 - \kappa \rho_M} \gamma_{M,t} - \frac{1 - \rho_m}{1 - \kappa \rho_m} \ln m_t + \frac{1 - \rho_\phi}{1 - \kappa \rho_\phi} \ln \phi_t \right) .\]

The unit discount factor implies the zero nominal interest rate. Perfectly flat money demand, i.e., the liquidity trap, at the steady state.

When $\kappa \rightarrow 1$, extremely large money disturbances are needed to explain the interest rate data.
Data

  - $\ln Y_t$: log differential of per capita output (nondurable consumption expenditure plus bilateral trade balance)
  - $\ln C_t$: log differential of per capita nondurable consumption expenditure
  - $\ln M_t$: log differential of per capita M1 money supply
  - $r_t$: differential of 3 month TB rates at quarterly rate.
  - $\ln S_t$: log of Canada-US bilateral nominal exchange rate.
Two prior specifications

▶ Benchmark model
  ▶ Prior of the subjective discount factor $\beta$ is uniformly distributed between 0 and 1.

▶ High Discount Factor (HDF) model
  ▶ Prior of the subjective discount factor $\beta$ has the Beta distribution with the mean of 0.999 and the S.D. of 0.001.

▶ Priors of the other parameters are shared by the two models to identify permanent and transitory components of exogenous impulses sharply.
## Prior distributions

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Distribution</th>
<th>Mean</th>
<th>S.D.</th>
<th>95 % Coverage</th>
</tr>
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<td>—</td>
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<td>Gamma</td>
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<td>0.850</td>
<td>0.100</td>
<td>[0.607 0.983]</td>
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</tbody>
</table>
Results: Benchmark model

- Discount factor $\kappa$ is distributed around 0.54, much smaller than the conventional size.
- The benchmark model fits to the actual exchange rate data fairly well.
- Money demand shock is almost RW with a large standard deviation.
- RW money demand shock dominates the FEVDs of the exchange rate.
## Exchange Rates and Fundamentals: A General Equilibrium Exploration

### Results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Benchmark Mean</th>
<th>Benchmark S.D.</th>
<th>HDF Mean</th>
<th>HDF S.D.</th>
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<td>0.041</td>
<td>0.950</td>
<td>0.001</td>
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<td>$\beta$</td>
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<td>0.042</td>
<td>0.998</td>
<td>0.000</td>
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<td>$\psi$</td>
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<td>0.006</td>
<td>0.000</td>
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<td>$\sigma_q$</td>
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<td>2148.572</td>
<td>1871.309</td>
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</table>
Actual exchange rate and in-sample predictions

(a) Benchmark Model

- Actual Exchange Rate
- Forecast
- 95% HPD

(b) Lower AR Roots of ln φ and Δln M^T

(c) HDF Model: Posterior Predictive Densities
## Forecast Error Variance Decompositions (\%): Benchmark Model

<table>
<thead>
<tr>
<th>horizon</th>
<th>$\epsilon_M$</th>
<th>$\epsilon_A$</th>
<th>$\epsilon_m$</th>
<th>$\epsilon_y$</th>
<th>$\epsilon_q$</th>
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<tr>
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<tr>
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<td>29</td>
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<td>0</td>
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<tr>
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<td>[1 3]</td>
<td>[60 70]</td>
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**Exchange Rate: $\ln S_t$**
Results: High Discount Factor Model

What can we find if we set the discount factor close to one within the benchmark model?

The unit discount factor sharply deteriorates the fit of the model with respect to two-country differentials in consumption and money supply.

1. Backus and Smith puzzle: almost perfect correlation between consumption differential and real exchange rate.

2. Flat money demand function
   - Needs a volatile money demand shock $\ln \phi_t$ to explain the interest rate differential.
   - Transitory exchange rate $\ln s_t$ becomes volatile.
   - Permanent money supply $\ln M^*_t$ should be volatile to keep the good fit to actual exchange rate $\ln S_t = \ln M^*_t + \ln s_t$
**FORECAST ERRORS TOWARDS DATA: BENCHMARK VS. HDF**

(a) Consumption Differential

(b) Money Supply Differential

(c) Exchange Rate

(d) Output Differential

(e) Interest Rate Differential
Results: High Discount Factor Model (cont’d)

- Bayesian posterior inference of the HDF model
  - Huge loss of overall fit to data relative to the Benchmark (marginal log likelihood: 1871.309 vs. 2148.572).
  - Counterfactually large volatilities of monetary disturbances.
  - Failure to predict actual money supply differential.
  - Backus and Smith puzzle: consumption differential is dominated by PPP deviation.
## Forecast Error Variance Decompositions (%): HDF Model

<table>
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<tr>
<th>horizon</th>
<th>$\epsilon_M$</th>
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<th>$\epsilon_m$</th>
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<td>[2 5]</td>
<td>[66 81]</td>
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</tr>
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</table>

**Consumption Differential: $\ln C_t$**

- $C_t$ represents the consumption differential.
Conclusion

- A canonical two-country model preserves the EW hypothesis of RW exchange rates.
- It is still a quite difficult task to explain RW exchange rates and economic fundamentals jointly and consistently within a canonical two-country incomplete-market model for Canada and the United States.
Where should we go?

- New Keynesian model with an I(1) economic fundamental
  - More realistic monetary policy framework: the Taylor rule
  - Endogenous real exchange rates
  - I(1) trend inflation

- Role of trend inflation in exchange rate dynamics: Kano (2016, HIAS and CAMA WP)
Thank you!