HIAS-E-80

Variable Competence and Collective Performance: Unanimity vs. Simple Majority Rule

Eyal Baharad[(a)], Ruth Ben-Yashar[(a)] and Shmuel Nitzan[(a),(b)]

(a) Department of Economics, Bar Ilan University, Ramat Gan 52900, Ierael
(b) Hitotsubashi Institute for Advanced Study, Hitotsubashi University

November, 2018

Hitotsubashi Institute for Advanced Study, Hitotsubashi University
2-1, Naka, Kunitachi, Tokyo 186-8601, Japan
tel:+81 42 580 8604 http://hias.ad.hit-u.ac.jp/

HIAS discussion papers can be downloaded without charge from:
http://hdl.handle.net/10086/27202
https://ideas.repec.org/s/hit/hiasdp.html

Variable Competence and Collective Performance:
Unanimity vs. Simple Majority Rule

by
Eyal Baharad [a], Ruth Ben-Yashar [a] and Shmuel Nitzan [b,c]

Abstract

Under the unanimity rule, a single voter may alter a decision that is unanimously accepted by all other voters. Under the simple majority rule, the impact of such a voter diminishes. This paper examines the marginal effect of competence on the collective performance – the likelihood of reaching a correct decision. It is shown that under the unanimity rule (simple majority rule), adding an incompetent voter to the group is inferior (superior) to giving up an existing competent voter. The negative impact of an incompetent voter cannot (can) always be balanced by adding a competent one when the decision mechanism is the unanimity (simple majority) rule. Moreover, improving a single voter's competence may have a greater effect on the collective performance under the simple majority rule relative to the unanimity rule.

Keywords: Unanimity rule, simple majority rule, voters' competence, collective performance.

[a] Department of Economics, Bar Ilan University, Ramat Gan 52900, Israel.
[b] Department of Economics, Bar Ilan University, Ramat Gan 52900, Israe and Hitotsubashi Institute for Advanced Study, Hitotsubashi University.
[c] The third author is grateful to Hitotsubashi Institute for Advanced Study at Hitotsubashi University for enabling the completion of this paper.

1. Introduction

Many collective decisions are based on the well-known simple majority or the unanimity rule. The former requires the support of a majority of the voters, while the latter is more demanding: a certain alternative is selected only if supported by all voters. Otherwise, the other alternative (possibly the status quo) is chosen.[1]

Unanimity rule and simple majority rule (SMR) are two polar examples of the qualified majority rules, under which a majority q, $½< q \leq 1$, of the voters, is required in order to approve a specific alternative. Somewhat surprisingly, these two polar cases are most common within the family of the qualified majority rules. Many organizations apply the unanimity rule for some specific decisions (usually sensitive ones) and the SMR for other issues. Some attempts have been made to bridge the gap between these two decision mechanisms; the recent one that we are aware of is by Bouton et al. (2018), who suggest a general framework of majority rule with veto power.

We focus our attention on the special, common cases of SMR and the unanimity rule examining the marginal effect of competence when these two mechanisms are applied. Voters' competence is a major factor that affects the likelihood of reaching a correct decision (assuming one exists). Intuitively, under the SMR every voter has a lesser impact on the collective outcome than under the unanimity rule (where every voter can change a decision that is unanimously accepted by all other voters).

Competence can be marginally increased by adding to the group one or two individuals or by increasing the skills of one of the existing group members. We assume that the a single voter, referred to as *incompetent* , whose decisional skills are equal to ½ (will choose the single correct alternative out of two given ones with a probability of ½) is added to a set of *competent* voters (each choosing the correct

---

[1] Earlier studies of majority rule include Baharad and Ben-Yashar (2009) who studied the validity of the optimal qualified majority rule under subjective probabilities. Berend and Sapir (2005) showed the superiority of the simple majority rule when decisional skills are unknown. Ladha (1995) relaxed the independence assumption under dichotomous choice. Nitzan and Paroush (1982) and Shapley and Grofman (1984) presented the conditions for the optimality of the majority rule. Studies focusing on the unanimity (hierarchy) rule include Ben-Yashar and Danziger (201**6**), Ben-Yashar and Nitzan (1998, 2001), Sah and Stiglitz (1988).

alternative with a probability greater than ½). The set of voters (before adding new ones) is homogeneous. Clearly, this is no longer the case after the set is expanded. The homogeneity assumption means that all voters have the same competence, i.e., they choose the correct alternative with a common probability.

Our first result establishes that eliminating a competent voter (possibly because of retirement) is better (worse) than adding an incompetent voter (typically, an unexperienced voter). Put differently, giving a chance to an incompetent decision-making-trainee is not (is) justified, even when he is escorted by a competent decision maker, when the decision mechanism is the unanimity rule (SMR). Our second result ascertains that an incompetent voter may have a (non) significant effect on the likelihood of reaching a correct decision, in the sense that his impact (can) cannot always be balanced by adding a competent one when the decision mechanism is the (SMR) unanimity rule. Moreover, as shown in our third result, the impact of improving a single voter's competence on this likelihood may be more substantial when the collective decision is made under the SMR (relative to the unanimity rule).

2. The model

Consider a case of two alternatives 1 and -1, one of which is correct and is thus better for all $n=2k+1$, $k>0$, voters.[2] As is common in decision problems, the identity of the better alternative is unknown. Every voter selects one of the two alternatives, and an aggregation mechanism is applied to select the collectively chosen alternative. The prior probability that each alternative is correct is ½.[3] Each voter chooses the correct alternative with probability p regardless of whether the correct alternative is 1 or -1. p reflects the voter's competence. We assume that $\forall\ j \neq i$, voter i's decision is independent of voter j's decision and $½< p <1$. The vector $\underline{p}^n = (p\ ,...,p\ )$ is referred to as the n voters' competence structure. Note that the homogeneity assumption means that the group of voters shares a common competence p.

---

[2] Earlier studies of two-alternative models include Ben-Yashar and Paroush (2000), Dietrich and List (2013) and Feld and Grofman (1984).

[3] This assumption implies unbiasedness and is especially plausible when analyzing jury's decisions. It is widely used; see, for example, Ben-Yashar and Danziger (2015) and Ben-Yashar and Nitzan (1997).

Let π denote the collective performance, namely, the probability that the group will choose the correct alternative. More specifically, $\pi_{SMR}\left(\underline{p}^n\right)$ denotes the probability that a group consisting of n voters associated with a competence vector $\underline{p}^n$ will choose the correct alternative under the SMR . Note that for the SMR, it is necessary for n to be an odd number . $\pi_{SMR}\left((p_h^{n-2}, ½, x)\right)$ denotes the probability that a group consisting of $n-2$ homogenous (h) voters associated with competence p, an incompetent voter with a competence of ½ and a competent one with a competence of x, will choose the correct alternative under the SMR.

Following Ben-Yashar and Paroush (2000), we present $\pi_{SMR}\left((p_h^{n-2}, ½, x)\right)$ as:

$$\pi_{SMR}\left((p_h^{n-2}, 1/2, x)\right) = 1/2\, xA + (1-(1-1/2)(1-x))B + C = 1/2\, xA + 1/2\,(x+1)B + C \quad (1)$$

where

$$A = \binom{2k-1}{k-1} p^{k-1}(1-p)^k,\ B = \binom{2k-1}{k} p^k (1-p)^{k-1} \qquad and$$

$$C = \sum_{g=k+1}^{2k-1} \binom{2k-1}{g} p^g (1-p)^{2k-1-g}$$

According to the unanimity rule, one of the alternatives (hereafter 1) is the selected outcome if and only if it is chosen by all n voters (otherwise the other alternative, -1, is chosen). Under this rule, let $\pi_{UNA}^{1}\left(\underline{p}^n\right)$ and $\pi_{UNA}^{-1}\left(\underline{p}^n\right)$ denote the probability of reaching a correct decision if alternative 1 and -1, respectively, is the correct one:[4]

$\pi_{UNA}^{1}\left(\underline{p}^n\right) = p^n$ and $\pi_{UNA}^{-1}\left(\underline{p}^n\right) = 1-(1-p)^n$.

[4] If the correct decision is 1 , the probability of a correct decision is equal to the probability that all the members support this alternative. If the correct alternative is -1, then the probability of a correct decision is equal to the probability that at least one member support -1.

$\pi_{UNA}\left(\underline{p}^n\right)$, the probability that a group consisting of n voters associated with a competence vector $\underline{p}^n$ will choose the correct alternative (each alternative with an equi-probable prior of 1/2) under the unanimity rule, is given by: [5]

$$\pi_{UNA}\left(\underline{P}^n\right)=\frac{\pi_{UNA}^{1}\left(\underline{p}^n\right)+\pi_{UNA}^{-1}\left(\underline{p}^n\right)}{2}. \quad (2)$$

$\pi_{UNA}((p_h^n, ½))$ denotes the performance of the group assuming that n homogenous voters have the decisional skill p and one voter the probability of ½.

Let $\pi_{UNA}\left((p_h^{n-2}, ½, x)\right)$ denote the probability that a group consisting of $n-2$ homogenous voters associated with a competence of p, an incompetent voter with a competence of ½ and a competent one with a competence of x, will choose the correct alternative under the unanimity rule. Note that for the unanimity rule, it is not necessary for n to be an odd number .

3. Giving up an existing competent voter or adding an incompetent voter

Eliminating a competent voter (possibly because of retirement) is better (worse) than adding an incompetent voter (typically, an unexperienced voter). Put differently, giving a chance to an incompetent decision-making-trainee is not (is) justified, even when he is escorted by a competent decision maker, when the decision mechanism is the unanimity rule (SMR). Under the simple majority rule with an odd number of voters, a meaningful increase in size requires the addition of at least two voters. Hence, under this rule, adding two unskilled voters to a group of experienced skilled voters is better than eliminating two of the existing skilled voters.

---

[5] Note that under the unanimity rule alternative 1 is selected if it is supported by all voters. Alternative -1 is selected if supported by at least one voter. Such asymmetry requires the distinction between $\pi_{UNA}^{1}\left(\underline{p}^n\right)$ and $\pi_{UNA}^{-1}\left(\underline{p}^n\right)$. Under the SMR, however, alternative 1 (or -1) is selected if supported by a simple majority of the voters. Such symmetry implies $\pi_{SMR}^{1}\left(\underline{p}^n\right)=\pi_{SMR}^{-1}\left(\underline{p}^n\right)$, which we denote as $\pi_{SMR}\left(\underline{p}^n\right)$.

Theorem 1: $\pi_{UNA}\left(\underline{p}^{2k-1}\right) > \pi_{UNA}\left((p_h^{2k}, ½\,)\right)$.

$$\pi_{SMR}\left(\underline{p}^{2k-1}\right) < \pi_{SMR}\left((p_h^{2k+1}, ½\,, ½)\right)$$

Proof:

The proof for the unanimity rule:

$$\pi_{UNA}\left((p_h^{2k}, ½\,)\right) = \frac{1}{2}\frac{1}{2}(p)^{2k} + \frac{1}{2}\left(1 - \frac{1}{2}(1-p)^{2k}\right) = \frac{1}{2} + \frac{1}{4}((p)^{2k} - (1-p)^{2k})$$

$$\pi_{UNA}\left(\underline{p}^{2k-1}\right) = \frac{1}{2}(p)^{2k-1} + \frac{1}{2}(1 - (1-p)^{2k-1}) = \frac{1}{2} + \frac{1}{2}((p)^{2k-1} - (1-p)^{2k-1})$$

Hence,

$$(p)^{2k-1} - (1-p)^{2k-1} > \frac{1}{2}((p)^{2k} - (1-p)^{2k}) \equiv$$

$$(p)^{2k-1}\left(1 - \frac{1}{2}p\ \right) > (1-p)^{2k-1}\left(1 - \frac{1}{2}(1-p)\ \right) \equiv$$

$$\left(\frac{p}{1-p}\right)^{2k-1} > \frac{1-\frac{1}{2}(1-p)}{1-\frac{1}{2}p}. \qquad (3)$$

If k=1, (3) is valid since:

$$\frac{p}{1-p} > \frac{1-\frac{1}{2}(1-p)}{1-\frac{1}{2}p} \equiv p - \frac{1}{2}p^2 > 1 - p - \frac{1}{2}(1-p)^2 \equiv 2(2p-1) > p^2 - (1-p)^2 \equiv$$

(3) is also valid for k>1 , since $2(2p-1) > (2p-1) \quad \equiv 2p - 1 > 0$, which is true. in the left hand size becomes larger and the right side of (3) is not changed.

The proof for SMR:

A straightforward application of Ben-Yashar and Zahavi (2011), implies that adding two incompetent voters to a group of competent voters is better than eliminating two of the existing competent voters.[6]

Q.E.D

Theorem 1 can be considered as a justification of supporting retirement of a single competent voter relative to preserving all competent voters and giving a chance to a single incompetent new voter, when the decision mechanism is the unanimity rule. More precisely, reducing the group size by eliminating one of the competent voters is

[6] Ben-Yashar and Zahavi (2011) showed that adding a pair of competent and incompetent voters to a group that consists of either competent or incompetent voters, provided that at least one of the existing group voters is incompetent, increases the probability that the group makes the correct decision. Under the unanimity rule, the result is preserved when comparing the case of adding two incompetent voters relative to eliminating two of the existing competent voters, see Appendix A**.** But we have chosen to present the more interesting case of adding just one competent voter relative to the case of eliminating a competent voter.

superior to increasing the group by a single incompetent voter. In contrast, under the simple majority rule, adding two incompetent voters to a group of competent voters is better than eliminating two of the existing competent voters. The case against retirement is therefore plausible under simple majority rule: adding two new inexperienced incompetent voters while preserving, instead of eliminating, two of the existing experienced competent voters enhances the performance of the group.

4. Balancing the impact of an incompetent voter

Let us now consider the case of balancing the impact of an incompetent voter. We examine the case where two voters are added to a set of *n*-2 homogenous voters. One of the added voters is associated with competence that is equal to ½, a voter referred to as an incompetent one. The other added voter is associated with competence *x*, who is intended to balance the incompetent voter, in the sense that the collective probability for reaching a correct decision is not reduced due to the addition of the two voters. We present the minimal required competence of the latter voter, when the two applied voting mechanisms are the SMR and the unanimity rules.[7]

Theorem 2:

$$\pi_{SMR}\left((p_h^{n-2}, ½, x)\right) \geq \pi_{SMR}\left(p_h^{n-2}\right) \Leftrightarrow x \geq p$$

$$\pi_{UNA}\left((p_h^{n-2}, ½, x)\right) \geq \pi_{UNA}\left(p_h^{n-2}\right) \Leftrightarrow \quad \frac{x+1}{2-x} \geq \left(\frac{p}{1-p}\right)^{n-2}$$

Proof:

By adding the two voters when the applied decision mechanism is the SMR, we obtain that, using equation (1):

$$\pi_{SMR}\left((p_h^{n-2}, ½, x)\right) = 0.5xA + B - 0.5(1-x)B + C$$

[7] The case of adding a member (a skilled voter with p>½) to an existing set of voters, was discussed in the literature (see, e.g., Feld and Grofman, 1984 or Karotkin and Paroush, 2003). Our study, like some recent studies (see, e.g., Ben-Yashar and Zahavi, 2011), considers the special case of adding a non-skilled voter (p=½).

$\pi_{SMR}(p_h^{n-2}) =$ B+C

Hence,

$$0.5\, xA + 0.5B(1+x) + C \geq B + C \equiv$$

$$0.5xA - 0.5(1-x)B \geq 0 \Leftrightarrow \frac{x}{1-x} \geq \frac{B}{A}$$

Since $\frac{B}{A} = \frac{p}{1-p}$, $x \geq p$.

By adding the two voters when the applied mechanism is the unanimity rule, we obtain that, using equation (2):

$$\frac{p^{n-2}0.5x + 1 - 0.5(1-x)(1-p)^{n-2}}{2} \geq \frac{p^{n-2} + 1 - (1-p)^{n-2}}{2} \Leftrightarrow$$

$$p^{n-2}(0.5x - 1) \geq (1-p)^{n-2}(0.5(1-x) - 1) \Leftrightarrow$$

$$p^{n-2}0.5(x-2) \geq -(1-p)^{n-2}0.5((1+x) \Leftrightarrow$$

$$\frac{p^{n-2}}{(1-p)^{n-2}} \leq \frac{1+x}{2-x}$$

from which we obtain that

$$\left(\frac{p}{1-p}\right)^{n-2} \leq \frac{x+1}{2-x}. \qquad (4)$$

Q.E.D.

Under the SMR, it is always possible to counterbalance the incompetent voter by adding one that is at least as competent as the voters in the group. It can be verified that under the unanimity rule, however, there are many cases where such balancing is impossible. Sufficient condition for such impossibility is obtained from (4), since x cannot exceed 1 and $\frac{x+1}{2-x}$ $increaes\ with\ x,$ $p > \frac{2^{1/(n-2)}}{1+2^{1/(n-2)}}$ (e.g., when p>2/3 and n>1,when p>0.56 and n>4, or when the group is large enough).

5. The impact of improving a single voter's competence

Our last objective is to examine the impact of improving a single voter's competence, when the applied rules are the SMR and the unanimity rule.

The first derivative of $\pi_{SMR}\left(\underline{p}^n\right)$ with respect to p_i(that is, the competence of a specific voter i) is obtained from (1), assuming that all the group voters are homogeneous. Recall that equation (1) views the probability of a correct collective decision by relating to the probabilities of two specific voters and to the probabilities of the remaining voters. The terms A,B and C do not depend on the probabilities of the two voters. Hence, when all voters are equally skilled and we change the competence of one of the specific voters i, equation (1) takes the form:

$p_i p\, A + \left(1-(1-p_i)(1-p)\right)B + C$ and, therefore,

$$\frac{\partial \pi_{SMR}\left(\underline{p}^n\right)}{\partial p_i} = pA + (1-p)B$$
,

which is positive, independent of p_i.

By taking the first derivative of $\pi_{UNA}\left(\underline{p}^n\right)$with respect to p_i, we obtain:

$$\frac{\partial \pi_{UNA}\left(\underline{p}^n\right)}{\partial p_i} = \frac{1}{2}\left(p^{n-1} + (1-p)^{n-1}\right).$$

which is positive as well, independent of p_i.

The following theorem reveals the condition under which changes in p_i are more influential under one rule relative to the other:

Theorem 3: Changes in p_i are more effective under SMR than under the unanimity rule when:

$$\frac{\partial \pi_{SMR}\left(\underline{P}^n\right)}{\partial p_i} > \frac{\partial \pi_{UNA}\left(\underline{P}^n\right)}{\partial p_i} \quad \text{if} \quad \left(\frac{p}{1-p}\right)^k < 2\cdot\binom{2k-1}{k-1} + \sqrt{4\cdot\binom{2k-1}{k-1}^2 - 1}$$

Proof:

$$\frac{\partial \pi_{SMR}\left(\underline{P}^n\right)}{\partial p_i} > \frac{\partial \pi_{UNA}\left(\underline{P}^n\right)}{\partial p_i} \Leftrightarrow pA+(1-p)B > \frac{1}{2}\left(p^{n-1}+(1-p)^{n-1}\right)$$

Since $n=2k+1$, we obtain:

$$2\cdot\binom{2k-1}{k-1}\cdot p^k(1-p)^k > \frac{1}{2}\left(p^{2k}+(1-p)^{2k}\right)$$

Hence,

$$0>\left(\frac{p}{1-p}\right)^{2k} - 4\cdot\binom{2k-1}{k-1}\cdot\left(\frac{p}{1-p}\right)^{k} + 1 \Leftrightarrow$$

$$2\cdot\binom{2k-1}{k-1} - \sqrt{4\binom{2k-1}{k-1}^2 - 1} < \left(\frac{p}{1-p}\right)^k < 2\cdot\binom{2k-1}{k-1} + \sqrt{4\binom{2k-1}{k-1}^2 - 1}$$

The left hand side condition is satisfied since, assuming $p>½$, $\left(\frac{p}{1-p}\right)^k > 1$ and $2\cdot\binom{2k-1}{k-1} - \sqrt{4\binom{2k-1}{k-1}^2 - 1} < 1$. Hence, the right hand side condition is identical to the one in the theorem.[8] Thus, the proof is completed.

Q.E.D.

Theorem 3 implies that changes in p_i are more effective under the SMR when p is small enough. In particular, this greater effectiveness is valid for k=1 when p<0.78, for k=2,3 when p<0.77 and for k=4 when p<0.75. This relates to jury's selection; jury applies the unanimity rule, under which a minor increase in relatively small p will not make a major change. Under committees (i.e., small number of members), however,

[8] Let u= $\binom{2k-1}{k-1}$. Then,

$$2\cdot u - \sqrt{4u^2-1} < 1 \Leftrightarrow 2\cdot u - 1 < \sqrt{4u^2-1} \Leftrightarrow$$
$$2u-1 < \sqrt{(2u-1)}\cdot\sqrt{(2u+1)} \Leftrightarrow \sqrt{(2u-1)} < \sqrt{(2u+1)},$$

which is always satisfied.

where the SMR is applied, a similar change in p would contribute significantly to the likelihood of reaching a correct decision. Theorem 3 characterizes the cases where improving decisional skills is useful.

6. Concluding remarks

This paper examines the marginal impact of competence on the collective likelihood of reaching a correct decision, when the decision mechanisms are the unanimity rule and the SMR. We first proved that, under the unanimity rule, increasing a homogeneous group of competent voters by a single incompetent voter is inferior to eliminating one of the existing competent group voters. This means that adding an incompetent voter accompanied by a competent one is disadvantageous. This is not the case under the simple majority rule, where the enlargement of a homogeneous group of capable voters by two balanced pairs of competent and incompetent voters enhances the performance of the group. In other words, under the simple majority rule, there exists an incentive to give a chance to incompetent voters-trainees as long as they are escorted by competent ones. Furthermore, we proved that the negative impact of an incompetent voter on the likelihood of reaching a correct decision cannot (can) always be balanced by adding a competent one when the decision mechanism is the unanimity rule (SMR). Moreover, the impact of improving a single voter's competence on this likelihood may be more substantial when the collective decision is made under the SMR (relative to the unanimity rule).

References

Baharad, E. and Ben-Yashar, R. (2009): "The Robustness of the Optimal Weighted Majority Rule to Probabilities Distortion", *Public Choice* 139, 53-59.

Ben-Yashar, R. and Danziger, L. (2015): "When is Voting Optimal", *Economic Theory Bulletin* 3, 341-356.

Ben-Yashar, R. and Danziger, L. (2016): "The Unanimity Rule and Extremely Asymmetric Committees", *Journal of Mathematical Economics* 64, 107-112.

Ben-Yashar, R., and Nitzan, S. (1997): "The Optimal Decision Rule for Fixed-Size Committees in Dichotomous Choice Situations: The General Result", *International Economic Review* 38(1): 175-186.

Ben-Yashar, R. and Nitzan, S. (1998): "Quality and Structure of Organizational Decision-Making", *Journal of Economic Behavior and Organization* 36, 521-534.

Ben-Yashar, R. and Nitzan, S. (2001): "The Robustness of Optimal Organizational Architectures: A Note on Hierarchies and Polyarchies", *Social Choice and Welfare* 18, 155-163.

Ben-Yashar, R. and Paroush, J. (2000): "A Nonasymptotic Condorcet Jury Theorem", *Social Choice and Welfare* 17, 189–199.

Ben-Yashar, R. and Zahavi, M. (2011): "The Condorcet Jury Theorem and Extension of the Franchise with Rationality Ignorant Voters", *Public Choice* 148, 435-443.

Berend, D. and Sapir, L. (2005): "Monotonicity in Condorcet Jury Theorem", *Social Choice and Welfare* 24, 83–92.

Bouton, L., Llorente-Saguer, A. and Malherbe, F. (2018): "Get Rid of Unanimity Rule: The Superiority of Majority Rules with Veto Power", *Journal of Political Economy* 126 (1), 107-149.

Dietrich, F. and List, C. (2013): "Propositionwise Judgment Aggregation: The General Case", *Social Choice and Welfare* 40, 1067-1095.

Feld, S.L. and Grofman, B. (1984): "The Accuracy of Group Majority Decisions in Groups with Added Members", *Public Choice* 42, 273–285.

Karotkin, D. and Paroush, J. (2003): "Optimum Committee Size: Quality – versus Quantity Dilemma", *Social Choice and Welfare* 20, 429–441.

Ladha, K.K. (1995): "Information Pooling through Majority-Rule Voting: Condorcet's Jury Theorem with Correlated Votes", *Journal of Economic Behavior and Organization* 26, 353-372.

Nitzan, S. and Paroush, J. (1982): "Optimal Decision Rules in Uncertain Dichotomous Choice Situation", *International Economic Review* 23, 289–297.

Sah, R.K. and Stiglitz, J.E. (1988): "Committees, Hierarchies and Polyarchies", *The Economic Journal* 98, 451-470.

Shapley, L. and Grofman, B. (1984): "Optimizing Group Judgmental Accuracy in the Presence of Interdependencies", *Public Choice* 43, 329–343.

Appendix A

$\pi_{UNA}\left(\underline{p}^{2k-1}\right) > \pi_{UNA}\left(\left(p_h^{2k+1}, ½ , ½\right)\right)$.

Proof:

$$\pi_{UNA}\left(\underline{p}^{2k-1}\right) = \frac{1}{2}(p)^{2k-1} + \frac{1}{2} - \frac{1}{2}(1-(1-p)^{2k-1}) =$$

$$\frac{1}{2} + \frac{1}{2}((p)^{2k-1} - (1-p)^{2k-1})$$

$$= \pi_{UNA}\left(\left(p_h^{2k+1}, ½ , ½\right)\right)$$

$$\frac{1}{2}\frac{1}{4}(p)^{2k+1} + \frac{1}{2} + \frac{1}{2}\left(1 - \frac{1}{4}(1-p)^{2k+1}\right))$$

Hence,

$$(p)^{2k-1} - (1-p)^{2k-1} > \frac{1}{4}((p)^{2k+1} - (1-p)^{2k+1}) \Leftrightarrow$$

$$(p)^{2k-1}\left(1 - \frac{1}{4}p^2\right) > (1-p)^{2k-1}\left(1 - \frac{1}{4}(1-p)^2\right) \Leftrightarrow$$

$$\frac{p}{1-p}^{2k-1} > \frac{1 - \frac{1}{4}(1-p)^2}{1 - \frac{1}{4}p^2}. \qquad (*)$$

(*) is valid since, if k=1,

$$\frac{p}{1-p} > \frac{1-\frac{1}{4}(1-p)^2}{1-\frac{1}{4}p^2} \equiv p - \frac{1}{4}p^3 > 1 - p - \frac{1}{4}(1-p)^3 \equiv 4(2p-1) > p^3 - (1-p)^3$$

(*) is also $\Leftrightarrow 4 > p^2 + p - p^2 + 1 - 2p + p^2 \equiv 0 > -3 - p + p^2, which\ is\ true.$
valid for k>1 , since in the left hand size becomes larger and the right side of (*) is not changed.

Q.E.D.