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Shutting-out-proofness in object allocation problems with money*

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Abstract

We study the problem of allocating heterogeneous objects to agents with money. Each agent can receive several objects and has a quasi-linear utility function. The owner of the objects is only interested in his revenue from an allocation. An (*allocation*) *rule* is *shutting-out-proof* if no group of agents together with the owner ever benefits from shutting out other groups of agents and arranging payments among themselves. We show that on any domain that includes all the additive valuation functions, a Vickrey rule satisfies *shutting-out-proofness* if and only if all the valuation functions in the domain satisfy the *substitutes condition* (Kelso and Crawford, 1982). Our result sheds a new light on the relationship between the desirable properties of a Vickrey rule and the substitutes condition (Ausubel and Milgrom, 2002; Milgrom, 2004).

JEL Classification Numbers. D44, D47, D71, D82

Keywords. Shutting-out-proofness, Collusion, Vickrey rules, Substitutes condition, Combinatorial auctions

1 Introduction

1.1 Purpose

We study the problem of allocating heterogeneous objects to agents with money. Each agent can receive several objects and has a quasi-linear utility function. A *valuation func-*

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tion specifies the valuation (the willingness to pay) of each set of objects. Note that a quasi-linear utility function is characterized by a valuation function. A class of valuation functions is called a *domain*. We consider the model where the set of agents can vary while the set of objects is fixed. An *economy* is a pair consisting of a set of agents and valuation functions of the agents. A (*consumption*) *bundle* is a pair of a set of objects that an agent receives and a payment that he makes. An *allocation* for a given economy specifies a bundle for each agent in the economy. We assume that the owner of the objects in only interested in his revenue (i.e., the sum of payments) from an allocation, and so he does not care who get what objects. An (*allocation*) *rule* is a function that associates an allocation with each economy.

Important examples of the problem are combinatorial auctions such as public procurement auctions, spectrum auctions, etc. One of the biggest concerns of practical auctions is to prevent agents from collusion (Klemperer, 2002). There is a plenty of research that studies collusion among a group of agents in auctions (Graham and Marshall, 1987; McAfee and McMillan, 1992; Pavlov, 2008; Che and Kim, 2009; Che et al., 2018; Shinozaki, 2022, etc.).

In contrast, to the best of our knowledge, there is no research prior to this paper and our companion papers (Shinozaki, 2023a,b) that studies rules preventing collusion between a group of agents *and the owner of the object*. In practical auctions such as public procurement auctions in Japan, collusion between a group of agents and the owner is pervasive as well as collusion among only a group of agents (McMillan, 1991, 2003). There are several types of such collusion, e.g., a group of agents together with the owner may benefit from misrepresenting the agents' valuation functions and arranging payments among themselves, or they may benefit from shutting out rival agents and arranging payments among themselves.

In this paper, we focus on collusion between a group of agents and the owner such that they benefit from shutting out other groups of agents and arranging payments among themselves.[1] Such collusion will suppress new entrants into a market, which is one of the biggest concerns of practical auction design as well as preventing collusion (Klemperer, 2002).

[1]In our companion papers (Shinozaki, 2023a,b), we study collusion such that a group of agents and the owner benefit from misrepresenting the agents' valuation functions and arranging payments among themselves.

In a practical auction, the planner (typically equivalent to the owner) is often able to select the participants of the auction, and so it is easy to conduct such collusion (McMillan, 1991, 2003). In fact, there are several examples of such collusion in the Japanese public procurement auctions that are detected and punished by the law. For example, in 2023, the former vice mayor of Kushima City, Miyazaki, Japan (corresponding to the owner), together with a president and employees of several construction companies (corresponding to a group of agents), was arrested for collusion such that they jointly shut out some group of companies in a procurement auction for public construction.[2]

The purpose of this paper is to study the class of rules that prevent a group of agents together with the owner from shutting out other groups of agents.

1.2 Main result

We propose a new property of a rule that prevents a set of agents and the owner from shutting out other agents. A rule satisfies *shutting-out-proofness* if, given an economy, no subset of the agents in the economy together with the owner ever benefits from shutting out other agents outside the economy and arranging payments among themselves. We also propose a new property that we call *aggregate utility monotonicity*. A rule satisfies *aggregate utility monotonicity* if, given an economy, the sum of utilities from an outcome allocation of the rule among each subset of the agents in the economy and the owner weakly increases when other agents entry into the economy. First, we establish the equivalence between *shutting-out-proofness* and *aggregate utility monotonicity* (Proposition 1): *On any domain, a rule satisfies shutting-out-proofness if and only if it satisfies aggregate utility monotonicity.*

We are interested in the class of rules satisfying *shutting-out-proofness* together with the other desirable and standard properties: *strategy-proofness*, *efficiency*, *individual rationality*, and *no subsidy*.[3] A rule is a *Vickrey rule* (Vickrey, 1961) if it allocates the objects to the agents so as to maximize the sum of valuations, and the payment of each agent is equal to the impact on the other agents. It is well known that the Vickrey rules are the

[2] A news article (in Japanese) that reports the collusion incident is: https://www3.nhk.or.jp/lnews/miyazaki/20231207/5060017062.html.

[3] A rule satisfies *strategy-proofness* if no agent ever benefits from misrepresenting his valuation functions. It satisfies *efficiency* if the object allocation chosen by the rule always maximizes the sum of valuations. It satisfies *individual rationality* if no agent ever gets worse off than the non-participation. It satisfies *no subsidy* if the payment of each agent is always non-negative.

only rules satisfying *strategy-proofness*, *efficiency*, *individual rationality*, and *no subsidy* (Holmstròm, 1979; Chew and Serizawa, 2007), Thus, we focus on the Vickrey rules, and study whether and when a Vickrey rule satisfies *shutting-out-proofness*.

A valuation function satisfies the *substitutes condition* (Kelso and Crawford, 1982) if the increase of a price of an object does not decrease the demand of the other objects. A valuation function is *additive* if it is an additive function. Note that each additive valuation function satisfies the substitutes condition.

The main result of this paper (Theorem 1) is as follows: *On any domain that includes all the additive valuation functions, a Vickrey rule satisfies shutting-out-proofness if and only if all the valuation functions in the domain satisfy the substitutes condition.* Recall that *shutting-out-proofness* is equivalent to *aggregate utility monotonicity* (Proposition 1). Thus, Theorem 1 also implies that a Vickrey rule on any domain that includes all the additive valuation functions satisfies *aggregate utility monotonicity* if and only if all the valuation functions in the domain satisfy the substitutes condition.

1.3 Related literature

There are several papers that study collusion only among agents. Some papers study the equilibrium properties of a specific rule such as a first-price and a second-price rules in the presence of collusion among agents (Graham and Marshall, 1987; McAfee and McMillan, 1992; Pesendorfer, 2000; Marshall and Marx, 2007, 2009, etc.), while others study rules that prevent collusion among agents (Pavlov, 2008; Che and Kim, 2009; Che et al., 2018; Shinozaki, 2022, etc.). This paper is close to the latter papers in that we also study rules that prevent collusion, but is different from those papers in that we focus on collusion between a group of agents and the owner, while they do collusion among a group of agents.

As already noted, there has been no previous study that investigates rules preventing collusion between a group of agents and the owner, and this paper together with our companion papers (Shinozaki, 2023a,b) are the first to study such rules. In contrast with this paper, Shinozaki (2023a,b) both study *two-sided collusion-proofness* which requires no group of agents together with the owner ever benefit from misrepresenting the agents' valuation functions and arranging payments among themselves. Shinozaki (2023b) establishes that the only rules satisfying *two-sided collusion-proofness*, *strategy-proofness*, *individual rationality*, and *no subsidy* are the *constant rules* which output the same outcome for each valuation profile, while a Vickrey rule can prevent specific two-sided collusion such as two-sided collusion without side payments and all-inclusive two-sided collusion. Shinozaki (2023a) characterizes the first-price rules in the single object model as the only rules that satisfy *Bayesian incentive compatibility*, *efficiency*, *individual rationality*, and *no subsidy*, and prevent "self-imposing" two-sided collusion under which no single agent has unilateral incentives to deviate from the collusion.

There are some papers that study corruption in auctions by the owner (Celentani and Ganuza, 2002; Compte et al., 2005; Menezes and Monteiro, 2006; Arozamena and Weinschelbaum, 2009, Lengwiler and Wolfstetter, 2010; Burguet, 2017, etc.) They study specific rules such as a first-price and a second-price rules or optimal rules in models where the owner has a chance to bribe agents, and investigate the equilibrium properties of specific rules or derive optimal rules. Note that collusion between a group of agents and the owner that we consider in this paper can be regarded as a kind of corruption by the owner. The difference between this paper and these paper lies in the attitude toward corruption. Indeed, we interpret corruption (i.e., collusion between a group of agents and the owner) as a phenomenon that should be prevented, and investigate rules that prevent such corruption. In contrast, they take the possibility of corruption as given, and study the equilibrium properties of rules or derive optimal rules in the presence of corruption.

It is known in the literature that a Vickrey rule on a domain including all the additive valuation functions satisfies the desirable properties such as *core allocation property*, *utility monotonicity*, *loser-collusion-proofness*, and *false-name-proofness* (Yokoo et al., 2004) if and only if all the valuation functions in the domain satisfy the substitutes condition (Ausubel and Milgrom, 2002; Ausubel, 2004).[4] Our main result (Theorem 1) sheds a new light on the relationship between the desirable properties of a Vickrey rule and the substitutes condition, i.e., the equivalence between *shutting-out-proofness* (or *aggregate utility monotonicity*) of a Vickrey rule and the substitutes condition.

[4]A rule satisfies *core allocation property* if an outcome allocation of a rule is always a core allocation. It satisfies *utility monotonicity* if the utility of each agent weakly increases when some agents leave from an economy. It satisfies *loser-collusion-proofness* if no group of losers ever benefits from misrepresenting their valuation functions. It satisfies *false-name-proofness* if no agent ever benefits from introducing "false-name" agents.

1.4 Organization

The rest of this paper is organized as follows. Section 2 introduces the model. Section 3 introduces the Vickrey rules. Section 4 presents the main result. Section 5 discusses in detail the relationships between *shutting-out-proofness* and the other desirable properties of a Vickrey rule under the substitutes condition. Section 6 concludes. All the proofs are relegated to the Appendix.

2 Model

The set of potential agents is $\mathbb{N}$. Let $\mathcal{N} = \{N \subseteq \mathbb{N} : 0 < |N| < \infty\}$. Let 0 indicate the owner of the objects. The owner holds $m \geq 1$ heterogeneous objects. Let $M = \{1, \ldots, m\}$ denote the set of objects. Let $\mathcal{M}$ denote the power set of M, i.e., $\mathcal{M} = 2^M$. A set of objects that an agent $i \in \mathbb{N}$ receives is $A_i \in \mathcal{M}$. The amount of a payment that an agent $i \in \mathbb{N}$ makes is $t_i \in \mathbb{R}$. Then, the (**consumption**) **set** of an agent $i \in \mathbb{N}$ is $\mathcal{M} \times \mathbb{R}$, and a (**consumption**) **bundle** of an agent $i \in \mathbb{N}$ is a pair $z_i = (A_i, t_i) \in \mathcal{M} \times \mathbb{R}$.

An agent $i \in \mathbb{N}$ has a quasi-linear utility function $u_i : \mathcal{M} \times \mathbb{R} \to \mathbb{R}$ such that for some **valuation function** $v_i : \mathcal{M} \to \mathbb{R}_+$, (i) $v_i(\varnothing) = 0$, (ii) for each pair $A_i, A_i' \in \mathcal{M}$ with $A_i \supseteq A_i'$, $v_i(A_i) \geq v_i(A_i')$, and (iii) for each $z_i = (A_i, t_i) \in \mathcal{M} \times \mathbb{R}$, $u_i(z_i; v_i) = v_i(A_i) - t_i$. The second condition (ii) above corresponds to free disposal. Note that a quasi-linear utility function is fully specified by a valuation function. Thus, we identify a set of quasi-linear utility functions with a set of valuation functions. Our generic notation for a class of valuation functions is $\mathcal{V}$. We call $\mathcal{V}$ a **domain**.

We introduce the two classes of valuation functions that will play important role in this paper.

Given $v_i \in \mathcal{V}$ and a *price vector* $p = (p^a)_{a \in M} \in \mathbb{R}_+^m$, the **demand set for** v_i **at** p is

$$D(v_i, p) = \Big\{A_i \in \mathcal{M} : \forall A_i' \in \mathcal{M},\ u_i\Big(\Big(A_i, \sum_{a \in A_i} p^a\Big); v_i\Big) \geq u_i\Big(\Big(A_i', \sum_{a \in A_i'} p^a\Big); v_i\Big)\Big\}.$$

First, a valuation function v_i satisfies the **substitutes condition** (Kelso and Crawford, 1982) if, for each price vector $p \in \mathbb{R}_+^m$, each $a \in M$, each $\varepsilon \in \mathbb{R}_{++}$, and each $A_i \in D(v_i, p)$, there is $A_i' \in D(v_i, p + \varepsilon \mathbf{e}^a)$ such that

$$A_i \cap (M \backslash \{a\}) \subseteq A_i',$$

where $\mathbf{e}^a$ denotes the a-th unit vector whose a-th component is 1, and all the other components are 0. Let $\mathcal{V}_{Sub}$ denote the class of valuation functions satisfying the substitutes condition. We call $\mathcal{V}_{Sub}$ the **substitutes domain**.

Second, a valuation function v_i is **additive** if it is an additive function, i.e., for each pair $A_i, A_i' \in \mathcal{M}$ with $A_i \cap A_i' = \varnothing$, we have $v_i(A_i \cup A_i') = v_i(A_i) + v_i(A_i')$. Let $\mathcal{V}_{Add}$ denote the class of additive valuation functions. We call $\mathcal{V}_{Add}$ the **additive domain**. Note that $\mathcal{V}_{Add} \subseteq \mathcal{V}_{Sub}$, i.e., each additive valuation function satisfies the substitutes condition. When $m = 1$, $\mathcal{V}_{Add} = \mathcal{V}_{Sub}$, and both the domains are equal to the class of all valuation functions.

Given $N \in \mathcal{N}$, a **valuation profile for** N is $v_N = (v_i)_{i \in N} \in \mathcal{V}^N$.

Given a domain $\mathcal{V}$, an **economy on** $\mathcal{V}$ is a triple $e = (N, M, v_N)$, where $N \in \mathcal{N}$ and $v_N \in \mathcal{V}^N$. The set of objects M are fixed throughout the paper, and so we may omit M from an economy. Thus, an economy is a pair $e = (N, v_N)$. Given $\mathcal{V}$, let $\mathcal{E}(\mathcal{V})$ denote the class of economies on $\mathcal{V}$. We may omit a domain $\mathcal{V}$ from $\mathcal{E}(\mathcal{V})$, and let $\mathcal{E}$ denote the class of all economies on $\mathcal{V}$.

Given $N \in \mathcal{N}$, a (**feasible**) **object allocation for** N is $A_N = (A_i)_{i \in N} \in \mathcal{M}^N$ such that $\cup_{i \in N} A_i \subseteq M$. Let X_N denote the set of all object allocations for N. Also, given $N \in \mathcal{N}$, a (**feasible**) **allocation for** N is $z_N = (z_i)_{i \in N} = (A_i, t_i)_{i \in N} \in (\mathcal{M} \times \mathbb{R})^n$ such that $(A_i)_{i \in N} \in X_N$. Let Z_N denote the set of all allocations for N. Given an allocation $z_N = (z_i)_{i \in N} \in Z_N$ for $N \in \mathcal{N}$ and $N' \in \mathcal{N}$ with $N' \subseteq N$, let $z_{N'} = (z_i)_{i \in N'}$.

The owner of the objects is only interested in his revenue from an allocation. Thus, he has a quasi-linear utility function u_0 from $\cup_{N \in \mathcal{N}} Z_N$ to $\mathbb{R}$ such that for each $N \in \mathcal{N}$ and each $z_N = (A_i, t_i)_{i \in N} \in Z_N$, $u_0(z_N) = \sum_{i \in N} t_i$.

An (**allocation**) **rule on** $\mathcal{V}$ is a function $f : \mathcal{E} \to \cup_{N \in \mathcal{N}} Z_N$ such that for each $e = (N, v_N) \in \mathcal{E}$, $f(e) \in Z_N$. Given a rule f on $\mathcal{V}$, let $x^f : \mathcal{E} \to \cup_{N \in \mathcal{N}} X_N$ denote the object allocation rule associated with f, and $t^f : \mathcal{E} \to \cup_{N \in \mathcal{N}} \mathbb{R}^N$ the associated payment rule. Given a rule f on $\mathcal{V}$, $e = (N, v_N) \in \mathcal{E}$, and $i \in N$, let $f_i(e) = (x_i^f(e), t_i^f(e))$ denote an outcome bundle of agent i for e under f.

We introduce the properties of rules.

The following is a new property which requires that given an economy $e = (N, v_N)$, no subset of N together with the owner ever benefit from shutting out other agents and arranging payments among themselves.

Shutting-out-proofness. For each pair $N, N' \in \mathcal{N}$ with $N \cap N' = \varnothing$ and each $v_{N\cup N'} \in \mathcal{V}^{N\cup N'}$, there are no $N'' \in \mathcal{N}$ with $N'' \subseteq N$ and $(p_i)_{i\in N''} \in \mathbb{R}^{N''}$ such that for each $i \in N''$,

$$u_i(f_i(N, v_N); v_i) - p_i > u_i(f_i(N \cup N', v_{N\cup N'}); v_i),$$

and

$$u_0(f(N, v_N)) + \sum_{i\in N''} p_i > u_0(f(N \cup N', v_{N\cup N'})).$$

The following is also a new property which requires that given an economy $e = (N, v_N)$, the sum of utilities from an outcome allocation of a rule among *each subset of* N and the owner weakly increase by the entry of other agents outside the economy.[5]

Aggregate utility monotonicity. For each pair $N, N' \in \mathcal{N}$ with $N \cap N' = \varnothing$, each $N'' \in \mathcal{N}$ with $N'' \subseteq N$, and each $v_{N\cup N'} \in \mathcal{V}^{N\cup N'}$, we have

$$\sum_{i\in N''} u_i(f_i(N \cup N', v_{N\cup N'}); v_i) + u_0(f(N \cup N', v_{N\cup N'})) \geq \sum_{i\in N''} u_i(f_i(e); v_i) + u_0(f(e)).$$

The following property requires that no agent ever benefit from misrepresenting his valuation functions.

Strategy-proofness. For each $e = (N, v_N) \in \mathcal{E}$, each $i \in N$, and each $v_i' \in \mathcal{V}$, we have $u_i(f_i(e); v_i) \geq u_i(f_i(N, (v_i', v_{N\setminus\{i\}})); v_i)$.

The following property requires that a rule select an allocation that maximize the sum

[5] Note that our *aggregate utility monotonicity* is different from the following natural monotonicity property of aggregate utility which requires that the aggregate utility weakly increase when a set of agents expands: For each pair $N, N' \in \mathcal{N}$ with $N \cap N' = \varnothing$ and each $v_{N\cup N'} \in \mathcal{V}^{N\cup N'}$, we have

$$\sum_{i\in N\cup N'} u_i(f_i(N \cup N', v_{N\cup N'}); v_i) + u_0(f(N \cup N', v_{N\cup N'})) \geq \sum_{i\in N} u_i(f_i(N, v_N); v_N) + u_0(f(N, v_N)).$$

In general, our *aggregate utility monotonicity* is independent of the above monotonicity property, i.e., the former does not necessarily imply the latter, and vice versa.

of valuations, i.e., a (*Pareto*) *efficient* allocation.

Efficiency. For each $e = (N, v_N) \in \mathcal{E}$, we have $x^f(e) \in \underset{x_N \in X_N}{\arg\max} \sum_{i \in N} v_i(x_i)$.

The following property requires that each agent find his outcome bundle of a rule at least as desirable as the non-participation under which he obtains the utility of zero.

Individual rationality. For each $e = (N, v_N) \in \mathcal{E}$ and each $i \in N$, $u_i(f_i(e); v_i) \geq 0$.

Finally, the following property requires that the payment of each agent be non-negative.

No subsidy. For each $e = (N, v_N) \in \mathcal{E}$ and each $i \in N$, $t_i^f(e) \geq 0$.

We are interested in the class of rules satisfying *shutting-out-proofness*, *strategy-proofness*, *efficiency*, *individual rationality*, and *no subsidy*, and study such a class of rules in this paper.

3 Vickrey rule

In this section, we introduce the Vickrey rules (Vickrey, 1961).

Given an economy $e = (N, v_N) \in \mathcal{E}$, let

$$w(e) = \max_{x_N \in X_N} \sum_{i \in N} v_i(x_i)$$

denote the maximal sum of valuations for the economy e. For notational convenience, let $w(\varnothing, v_\varnothing) = 0$. Note that a rule f on $\mathcal{V}$ is *efficient* if and only if for each $e = (N, v_N) \in \mathcal{E}$, $w(e) = \sum_{i \in N} v_i(x_i^f(e))$.

Under a Vickrey rule, the objects are allocated so as to maximize the sum of valuations, and each agent pays the impact on the other agents.

Definition 1. A rule f on $\mathcal{V}$ is a **Vickrey rule** (Vickrey, 1961) if, for each $e = (N, v_N) \in \mathcal{E}$,

$$x^f(e) \in \underset{x_N \in X_N}{\arg\max} \sum_{i \in N} v_i(x_i),$$

and for each $i \in N$

$$t_i^f(e) = w(N\backslash\{i\}, v_{N\backslash\{i\}}) - \sum_{j \in N\backslash\{i\}} v_j(x_j^f(e)).$$

The following fact states that on any domain that includes the additive domain, the Vickrey rules are the only rules satisfying *strategy-proofness*, *efficiency*, *individual rationality*, and *no subsidy*.

Fact 1 (Holmstròm, 1979; Chew and Serizawa, 2007). *Let $\mathcal{V}$ be a domain such that $\mathcal{V} \supseteq \mathcal{V}_{Add}$. A rule on $\mathcal{V}$ satisfies strategy-proofness, efficiency, individual rationality, and no subsidy if and only if it is a Vickrey rule.*

Thanks to Fact 1, we can restrict our attention to the class of Vickrey rules. Then, we study whether and when a Vickrey rule satisfies *shutting-out-proofness* in the next section.

4 Main result

In this section, we present the main result of this paper.

First, the next result states that a rule on any domain satisfies *shutting-out-proofness* if and only if it satisfies *aggregate utility monotonicity*. That is, *shutting-out-proofness* is equivalent to *aggregate utility monotonicity* on any domain.

Proposition 1. *Let $\mathcal{V}$ be a domain. Let f be a rule on $\mathcal{V}$. The following two statements are equivalent.*
(i) f satisfies shutting-out-proofness.
(ii) f satisfies aggregate utility monotonicity.

The next result is the main result of this paper which states that a rule on any domain that includes the additive domain satisfies *shutting-out-proofness* (or equivalently, *aggregate utility monotonicity* by Proposition 1) if and only if all the valuation functions in the domain satisfy the substitutes condition.

Theorem 1. *Let $\mathcal{V}$ be a domain such that $\mathcal{V} \supseteq \mathcal{V}_{Add}$. Let f be a Vickrey rule on $\mathcal{V}$. The following three statements are equivalent.*
(i) f satisfies shutting-out-proofness.

(ii) f satisfies aggregate utility monotonicity.

(iii) We have $\mathcal{V} \subseteq \mathcal{V}_{Sub}$.

A corollary of Theorem 1 and Fact 1 is that the substitutes domain is the unique maximal domain that includes the additive domain for the existence of a rule satisfying *shutting-out-proofness* (or *aggregate utility monotonicity*), *strategy-proofness*, *efficiency*, *individual rationality*, and *no subsidy*.

Corollary 1. *Let $\mathcal{V}$ be a domain such that $\mathcal{V} \supseteq \mathcal{V}_{Add}$. The following three statements are equivalent.*

(i) There is a rule on $\mathcal{V}$ satisfying shutting-out-proofness, strategy-proofness, efficiency, individual rationality, and no subsidy.

(ii) There is a rule on $\mathcal{V}$ satisfying aggregate utility monotonicity, strategy-proofness, efficiency, individual rationality, and no subsidy.

(iii) We have $\mathcal{V} \subseteq \mathcal{V}_{Sub}$.

5 Discussion

In this section, we discuss the relationships between *shutting-out-proofness* and the other desirable properties of a Vickrey rule under the substitutes condition.

First, we introduce the properties that a Vickrey rule satisfies under the substitutes condition.

Given $e = (N, v_N) \in \mathcal{E}$, an allocation $z_N = (z_i)_{i \in N} = (x_i, t_i)_{i \in N} \in Z$ for N is a **core allocation for** e if $w(N) = \sum_{i \in N} v_i(x_i)$, and there exists no $N' \in \mathcal{N}$ with $N' \subseteq N$ such that $w(N') > \sum_{i \in N'} u_i(z_i; v_i) + u_0(z_{N'})$. The following property requires that an outcome allocation of a rule be a core allocation.

Core allocation property. For each $e \in \mathcal{E}$, $f(e)$ is a core allocation for e.

The next property requires that each agent get weakly better off when some agents leave from the economy.

Utility monotonicity. For each $e = (N, v_N) \in \mathcal{E}$, each $N' \in \mathcal{N}$ with $N' \subseteq N$, and each

$i \in N'$, we have

$$u_i(f_i(N', v_{N'}); v_i) \geq u_i(f_i(e); v_i).$$

The next property requires that no group of losers who receive no object ever benefit from misrepresenting their valuation functions.

Loser-collusion-proofness. For each $e = (N, v_N)$ and each $N' \in \mathcal{N}$ with $N' \subsetneq N$ and $A_i^f(e) = \varnothing$ for each $i \in N'$, there is no $v'_{N'} \in \mathcal{V}^{N'}$ such that for each $i \in N'$,

$$u_i(f_i(N, (v'_{N'}, v_{N \setminus N'})); v_i) > u_i(f_i(e); v_i) \tag{1}$$

Finally, the following property was introduced by Yokoo et al. (2004), which requires that no agent ever benefit from introducing "false-name" agents.

False-name-proofness. For each $e = (N, v_N) \in \mathcal{E}$ and each $N' \in \mathcal{N}$ with $N \cap N' = \varnothing$, there is no $v_{N'} \in \mathcal{V}^{N'}$ such that

$$v_i\Big(\bigcup\nolimits_{j \in N' \cup \{i\}} A_j^f(N \cup N', v_{N \cup N'})\Big) - \sum_{j \in N' \cup \{i\}} t_j^f(N \cup N', v_{N \cup N'}) > u_i(f_i(e); v_i).$$

The following fact states that a Vickrey rule on any domain that includes the additive domain satisfies any of the above four properties if and only if all the valuation functions in the domain satisfy the substitutes condition. Thus, under a Vickrey rule, the above four properties are all equivalent.

Fact 2 (Ausubel and Milgrom, 2002; Milgrom, 2004). *Let $\mathcal{V}$ be a domain such that $\mathcal{V} \supseteq \mathcal{V}_{Add}$. Let f be a Vickrey rule on $\mathcal{V}$. The following five statements are equivalent.*

(i) f satisfies core allocation property.

(ii) f satisfies utility monotonicity.

(iii) f satisfies loser-collusion-proofness.

(iv) f satisfies false-name-proofness.

(v) We have $\mathcal{V} \subseteq \mathcal{V}_{Sub}$.

Our result (Theorem 1) adds a new property to the list of desirable properties of a Vickrey rule under the substitutes condition in Fact 2: *shutting-out-proofness* (or equivalently, *aggregate utility monotonicity*). By Theorem 1 and Fact 2, *shutting-out-proofness* and *aggregate utility monotonicity* are both equivalent to any of the above four properties under a Vickrey rule. By Fact 1, we can also conclude that *shutting-out-proofness* (or *aggregate utility monotonicity*) is equivalent to any of the four properties under a rule satisfying *strategy-proofness*, *efficiency*, *individual rationality*, and *no subsidy*.

In the remaining part of this section, we show that if we drop either *strategy-proofness* or *efficiency*, then *shutting-out-proofness* (or *aggregate utility monotonicity*) is no longer equivalent to any of the above four properties. Thus, *shutting-out-proofness* (or *aggregate utility monotonicity*) itself is an independent property of the other desirable properties of a Vickrey rule under the substitutes condition.

First, we demonstrate that if we drop *strategy-proofness*, then *shutting-out-proofness* is independent of *core allocation property*, i.e., *shutting-out-proofness* does not necessarily imply *core allocation property*, and vice versa.

Example 1 (Core allocation property is independent of shutting-out-proofness). Let $m = 1$. Let $\mathcal{V} = \mathcal{V}_{Sub}$.

(i) Let f be a third-price rule on $\mathcal{V}$.[6] Then, it satisfies *shutting-out-proofness*, but violates *core allocation property*. Note that it also satisfies *efficiency*, *individual rationality* and *no subsidy*, but violates *strategy-proofness*.

(ii) Let f be a rule on $\mathcal{V}$ such that for each $e = (N, v_N) \in \mathcal{E}$, if $|N| = 1$, then $f(e)$ is equivalent to the outcome of a first-price rule for e,[7] and otherwise, it is equivalent to the outcome of a second-price rule for e.[8] Then, it satisfies *core allocation property*, but violates *shutting-out-proofness*. Note that it also satisfies *efficiency*, *individual rationality*, and *no subsidy*, but violates *strategy-proofness*. □

The next example shows that if we drop *efficiency*, then *utility monotonicity* is independent of *shutting-out-proofness*.

Example 2 (Utility monotonicity is independent of shutting-out-proofness). Let $m = 1$. Let $\mathcal{V} = \mathcal{V}_{Sub}$.

[6] A rule f on $\mathcal{V}$ is a *third-price rule* if for each $e = (N, v_N) \in \mathcal{E}$, an agent with the highest valuation wins the object, he pays the third-highest valuation (if $|N| \leq 2$, then he pays nothing), and each loser pays nothing.

[7] A rule f on $\mathcal{V}$ is a *first-price rule* if for each $e \in \mathcal{E}$, an agent with the highest valuation wins the object, he pays his own valuation, and each loser pays nothing.

[8] A rule f on $\mathcal{V}$ is a *second-price rule* if an agent with the highest valuation wins the object, he pays the second-highest valuation (if $|N| = 1$, then he pays nothing), and each loser pays nothing. Note that a second-price rule is equivalent to a Vickrey rule in the single object setting.

(i) Let f be a rule on $\mathcal{V}$ such that for each $e = (N, v_N) \in \mathcal{E}$, if $|N| = 1$, then $f(e)$ is equivalent to the outcome of the no-trade rule for e,[9] and otherwise, it is equivalent to the outcome of a second-price rule for e. Then, it satisfies *shutting-out-proofness*, but violates *false-name-proofness*. Note that it also satisfies *strategy-proofness*, *individual rationality*, and *no subsidy*, but violates *efficiency*.

(ii) Let f be a rule on $\mathcal{V}$ such that for each $e = (N, v_N) \in \mathcal{E}$, if $|N| = 1$, then $f(e)$ is equivalent to the outcome of a second-price rule for e, and otherwise, it is equivalent to the outcome of the no-trade rule for e. Then, it satisfies *false-name-proofness*, but violates *shutting-out-proofness*. Note that it also satisfies *strategy-proofness*, *individual rationality*, and *no subsidy*, but violates *efficiency*. □

The next example shows that if we drop *strategy-proofness*, then *loser-collusion-proofness* is independent of *shutting-out-proofness*.

Example 3 (Loser-collusion-proofness is independent of shutting-out-proofness). Let $m = 1$. Let $\mathcal{V} = \mathcal{V}_{Sub}$.

(i) Let f be a rule on $\mathcal{V}$ such that for each $e = (N, v_N) \in \mathcal{E}$, if $|N| \leq 2$, then $f(e)$ is equivalent to the outcome of a third-price rule, and otherwise, it is equivalent to the outcome of a second-price rule. Then, it satisfies *shutting-out-proofness*, but violates *loser-collusion-proofness*. It also satisfies *efficiency*, *individual rationality* and *no subsidy*, but violates *strategy-proofness*.

(ii) Let f be a rule on $\mathcal{V}$ such that for each $e = (N, v_N) \in \mathcal{E}$, if $|N| \leq 2$, then $f(e)$ is equivalent to the outcome of a first-price rule for e, and otherwise, it is equivalent to the outcome of a second-price rule for e. Then, it satisfies *loser-collusion-proofness*, but violates *shutting-out-proofness*. It also satisfies *efficiency*, *individual rationality*, and *no subsidy*, but violates *strategy-proofness*. □

Finally, the next example shows that if we drop *efficiency*, then *false-name-proofness* is independent of *shutting-out-proofness*.

Example 4 (False-name-proofness is independent of shutting-out-proofness). Let $m = 1$. Let $\mathcal{V} = \mathcal{V}_{Sub}$.

(i) Let f be a rule on $\mathcal{V}$ defined in Example 2 (i). Then, it satisfies *shutting-out-proofness*,

[9] A rule f on $\mathcal{V}$ is the *no-trade rule* if for each $e = (N, v_N) \in \mathcal{E}$ and each $i \in N$, $f_i(e) = (\varnothing, 0)$.

but violates *false-name-proofness*. It also satisfies *strategy-proofness*, *individual rationality*, and *no subsidy*, but violates *efficiency*.
(ii) Let f be a rule on $\mathcal{V}$ defined in Example 2 (ii). Then, it satisfies *false-name-proofness*, but violates *shutting-out-proofness*. It also satisfies *strategy-proofness*, *individual rationality*, and *no subsidy*, but violates *efficiency*. □

6 Conclusion

In this paper, we have considered collusion between a group of agents and the owner such that they benefit from shutting out other groups of agents with the possibility of arranging payments among themselves. We have proposed a new property of a rule that prevents such collusion (*shutting-out-proofness*) , and established that a Vickrey rule on a domain that includes the additive domain satisfies *shutting-out-proofness* if and only if all the valuation functions in the domain satisfy the substitutes condition (Theorem 1). Our result provides a new insight into the relationship between the desirable properties of a Vickrey rule and the substitutes condition (Ausubel and Milgrom, 2002; Milgrom, 2004).

In our formulation of *shutting-out-proofness*, we allow a group of agents and the owner to arrange payments among themselves. Such a "bribe" (i.e., side payments) is typical in practical collusion between agents and the owner,[10] but it is also possible that they may not arrange payments in fear of detection of the collusion by the antitrust authority. Then, it would be interesting to investigate when a Vickrey rule satisfies a weaker version of *shutting-out-proofness* without the possibility of arranging payments among themselves.

Appendix

A Proof of Proposition 1

In this section, we prove Proposition 1.

[10]For example, a news article reports that the former mayor of Shika Town, Ishikawa, Japan, and the president of a construction company in Shika Town were arrested for leakage of the minimum bid price from the former mayor and a bribe from the president of a construction company in a Japanese procurement auction for public construction. See: https://www.asahi.com/articles/ASRD902DKRD8PISC00S.html.

A.1 (i) implies (ii)

First, we show that (i) implies (ii). We show the contrapositive. Suppose that f violates *aggregate utility monotonicity*. Then, there exist a pair $N, N' \in \mathcal{N}$ with $N \cap N' = \varnothing$, $N'' \in \mathcal{N}$ with $N'' \subseteq N$, and $v_{N\cup N'} \in \mathcal{V}^{N\cup N'}$ such that

$$\sum_{i\in N''} u_i(f_i(N, v_N); v_i) + u_0(f(N, v_N)) > \sum_{i\in N''} u_i(f_i(N\cup N', v_{N\cup N'}); v_i) + u_0(f(N\cup N', v_{N\cup N'})).$$

Then, we can choose $\varepsilon \in \mathbb{R}_{++}$ such that

$$|N''|\varepsilon < u_0(f(N, v_N)) + \sum_{i\in N''} \Big(u_i(f_i(N, v_N); v_i) - u_i(f_i(N\cup N', v_{N\cup N'}); v_i) \Big) - u_0(f(N\cup N', v_{N\cup N'})). \quad (1)$$

For each $i \in N''$, let $p_i = u_i(f_i(N, v_N); v_i) - u_i(f_i(N\cup N', v_{N\cup N'}); v_i) - \varepsilon$. Then, for each $i \in N''$, by $\varepsilon > 0$,

$$u_i(f_i(N, v_N); v_i) - p_i = u_i(f_i(N\cup N', v_{N\cup N'}); v_i) + \varepsilon > u_i(f_i(N\cup N', v_{N\cup N'}); v_i).$$

We also have

$$\begin{aligned} & u_0(f(N, v_N)) + \sum_{i\in N''} p_i - u_0(f(N\cup N', v_{N\cup N'})) \\ = {} & u_0(f(N, v_N)) + \sum_{i\in N''} \Big(u_i(f_i(N, v_N); v_i) - u_i(f_i(N\cup N', v_{N\cup N'}); v_i) \Big) - u_0(f(N\cup N', v_{N\cup N'})) - |N'|\varepsilon \\ > {} & 0, \end{aligned}$$

where the last inequality follows from (1). Thus, f violates *shutting-out-proofness*. ■

A.2 (ii) implies (i)

We show that (ii) implies (i). Again, we show the contrapositive. Suppose that f violates *shutting-out-proofness*. Then, there exist a pair $N, N' \in \mathcal{N}$ with $N \cap N' = \varnothing$, $N'' \in \mathcal{N}$ with $N'' \subseteq N$, $v_{N\cup N'} \in \mathcal{V}^{N\cup N'}$, and $(p_i)_{i\in N''} \in \mathbb{R}^{N''}$ such that for each $i \in N''$,

$$u_i(f_i(N, v_N); v_i) - p_i > u_i(f_i(N\cup N', v_{N\cup N'}); v_i),$$

and

$$u_0(f(N, v_N)) + \sum_{i \in N''} p_i > u_0(f(N \cup N', v_{N \cup N'})).$$

Summing up these inequalities yield

$$\begin{aligned}
&\sum_{i \in N''} u_i(f_i(N, v_N); v_i) + u_0(f(N, v_N)) \\
&= \sum_{i \in N''} \Big(u_i(f_i(N, v_N); v_i) - p_i \Big) + u_0(f(N, v_N)) + \sum_{i \in N''} p_i \\
&> \sum_{i \in N''} u_i(f_i(N \cup N', v_{N \cup N'}); v_i) + u_0(f(N \cup N', v_{N \cup N'})).
\end{aligned}$$

Thus, f violates *aggregate utility monotonicity*. ■

B Proof of Theorem 1

In this section, we prove Theorem 1. Note that Proposition 1 implies that (i) is equivalent to (ii). Thus, it suffices to show that (ii) implies (iii) and (iii) implies (ii).

We begin with the following lemma. Although it follows from Theorem 8.1 of Milgrom (2004), we give a self-standing proof for completeness.

Lemma 1. *For each $e = (N, v_N) \in \mathcal{E}$ and each $N' \in \mathcal{N}$ with $N' \subseteq N$, we have*

$$\sum_{i \in N'} u_i(f_i(e); v_i) + u_0(f(e)) = w(e) - \sum_{i \in N \setminus N'} \Big(w(e) - w(N \setminus \{i\}, v_{N \setminus \{i\}}) \Big).$$

Proof. For each $i \in N$, we have

$$t_i^f(e) = w(N \setminus \{i\}, v_{N \setminus \{i\}}) - \sum_{j \in N \setminus \{i\}} v_j(x_j^f(e)) = w(N \setminus \{i\}, v_{N \setminus \{i\}}) - (w(e) - v_i(x_i^f(e))), \quad (1)$$

where the second equality follows from *efficiency* (i.e., $w(e) = \sum_{j \in N} v_j(x_j^f(e))$). Then,

$$\begin{aligned}
u_0(f(e)) &= \sum_{i \in N} t_i^f(e) = \sum_{i \in N} \Big(w(N \setminus \{i\}, v_{N \setminus \{i\}}) - (w(e) - v_i(x_i^f(e))) \Big) \\
&= \sum_{i \in N} \Big(w(N \setminus \{i\}, v_{N \setminus \{i\}}) - w(e) \Big) + \sum_{i \in N} v_i(x_i^f(e)) \\
&= w(e) - \sum_{i \in N} \Big(w(e) - w(N \setminus \{i\}, v_{N \setminus \{i\}}) \Big), \quad (2)
\end{aligned}$$

where the second equality follows from (1), and the last one from *efficiency*. Also, for each $i \in N$, we have

$$u_i(f_i(e); v_i) = v_i(x_i^f(v)) - \Big(w(N\backslash\{i\}, v_{N\backslash\{i\}}) - \sum_{j \in N\backslash\{i\}} v_j(x_j^f(e))\Big)$$
$$= \sum_{j \in N} v_j(x_j^f(e)) - w(N\backslash\{i\}, v_{N\backslash\{i\}}) = w(e) - w(N\backslash\{i\}, v_{N\backslash\{i\}}), \quad (3)$$

where the last equality follows from *efficiency*. Then,

$$\sum_{i \in N'} u_i(f_i(e); v_i) + u_0(f(e))$$
$$= w(e) - \sum_{i \in N}\Big(w(e) - w(N\backslash\{i\}, v_{N\backslash\{i\}})\Big) + \sum_{i \in N'}\Big(w(e) - w(N\backslash\{i\}, v_{N\backslash\{i\}})\Big) \quad \text{(by (2) and 3)}$$
$$= w(e) - \sum_{i \in N\backslash N'}\Big(w(e) - w(N\backslash\{i\}, v_{N\backslash\{i\}})\Big),$$

as desired. □

Given a domain $\mathcal{V}$, w satisfies **bidder submodularity on** $\mathcal{V}$ if for each $e = (N, v_N) \in \mathcal{E}$ with $v_N \in \mathcal{V}^N$, each $N' \in \mathcal{N}$ with $N' \subseteq N$, and each $i \in N'$, we have

$$w(N', v_{N'}) - w(N'\backslash\{i\}, v_{N'\backslash\{i\}}) \geq w(N, v_N) - w(N\backslash\{i\}, v_{N\backslash\{i\}}).$$

We invoke the following fact which states that on any domain $\mathcal{V}$ that includes the additive domain, w satisfies bidder submodularity on $\mathcal{V}$ if and only if the domain $\mathcal{V}$ is included in the substitutes domain.

Fact 3 (Ausubel and Milgrom, 2002; Milgrom, 2004). *Let $\mathcal{V}$ be a domain such that $\mathcal{V} \supseteq \mathcal{V}_{Add}$. The following two statement are equivalent.*
(i) w satisfies bidder submodularity on $\mathcal{V}$.
(ii) We have $\mathcal{V} \subseteq \mathcal{V}_{Sub}$

B.1 (ii) implies (iii)

We show that (ii) implies (iii). We prove the contrapositive. Suppose that (iii) does not hold, i.e., $\mathcal{V} \not\subseteq \mathcal{V}_{Sub}$. Then, by Fact 2, f violates *core allocation property*. Then, by *efficiency* of f, there exist a pair $N, N' \in \mathcal{N}$ with $N' \subseteq N$ and $v_N \in \mathcal{V}^N$ such that

$$w(N', v_{N'}) > \sum_{i \in N'} u_i(f_i(N, v_N); v_i) + u_0(f(N, v_N)). \quad (1)$$

By Lemma 1, we have

$$\sum_{i \in N} u_i(f_i(N, v_N); v_i) + u_0(f(N, v_N)) = w(N, v_N).$$

Thus, by (1), we must have $N' \neq N$. Thus, by $N' \subseteq N$, $N' \subsetneq N$. Also, by Lemma 1, we have

$$\sum_{i \in N'} u_i(f_i(N', v_{N'}); v_i) + u_0(f(N', v_{N'})) = w(N', v_{N'}). \quad (2)$$

Combining (1) and (2), we get

$$\sum_{i \in N'} u_i(f_i(N', v_{N'}); v_i) + u_0(f(N', v_{N'})) = w(N', v_{N'}) > \sum_{i \in N'} u_i(f_i(N, v_N); v_i) + u_0(f(N, v_N)).$$

Thus, by $N' \subsetneq N$, f violates *aggregate utility monotonicity*, i.e., (ii) does not hold. ■

B.2 (iii) implies (ii)

We show that (iii) implies (ii). Suppose (iii) holds, i.e., $\mathcal{V} \subseteq \mathcal{V}_{Sub}$. By Fact 3, w satisfies bidder submodularity on $\mathcal{V}$. Let $N, N' \in \mathcal{N}$ be a pair such that $N \cap N' = \varnothing$. Let $N'' \in \mathcal{N}$ be such that $N'' \subseteq N$. Let $v_{N \cup N'} \in \mathcal{V}^{N \cup N'}$. By bidder submodularity on $\mathcal{V}$, we have

$$\begin{aligned} &\sum_{i \in N \backslash N''} \Big(w(N, v_N) - w(N \backslash \{i\}, v_{N \backslash \{i\}}) \Big) \\ \geq &\sum_{i \in N \backslash N''} \Big(w(N \cup N', v_{N \cup N'}) - w(N \cup N' \backslash \{i\}, v_{N \cup N' \backslash \{i\}}) \Big). \end{aligned} \quad (3)$$

Let $N' = \{i_1, \ldots, i_K\}$. For each $k \in \{1, \ldots, K\}$, let $N'(k) = \{i_1, \ldots, i_k\}$. For notational convenience, let $N'(0) = \varnothing$. We have

$$\begin{aligned} w(N \cup N', v_{N\cup N'}) - w(N, v_N) &= \sum_{k=1}^{K} \Big(w(N \cup N'(k), v_{N\cup N'(k)}) - w(N \cup N'(k-1), v_{N\cup N'(k-1)}) \Big) \\ &\geq \sum_{k=1}^{K} \Big(w(N \cup N', v_{N\cup N'}) - w(N \cup N' \backslash \{i_k\}, v_{N\cup N'\backslash\{i_k\}}) \Big) \\ &= \sum_{i\in N'} \Big(w(N \cup N', v_{N\cup N'}) - w(N \cup N' \backslash \{i\}, v_{N\cup N'\backslash\{i\}}) \Big), \end{aligned} \tag{4}$$

where the inequality follows from bidder submodularity on $\mathcal{V}$. Then, we have

$$\begin{aligned} & w(N \cup N', v_{N\cup N'}) - \sum_{i\in N\cup N'\backslash N''} \Big(w(N \cup N', v_{N\cup N'}) - w(N \cup N' \backslash \{i\}, v_{N\cup N'\backslash\{i\}}) \Big) \\ &= w(N \cup N', v_{N\cup N'}) - \sum_{i\in N\backslash N''} \Big(w(N \cup N', v_{N\cup N'}) - w(N \cup N' \backslash \{i\}, v_{N\cup N'\backslash\{i\}}) \Big) \\ &\quad - \sum_{i\in N'} \Big(w(N \cup N', v_{N\cup N'}) - w(N \cup N' \backslash \{i\}, v_{N\cup N'\backslash\{i\}}) \Big) \\ &\geq w(N \cup N', v_{N\cup N'}) - \sum_{i\in N\backslash N''} \Big(w(N, v_N) - w(N \backslash \{i\}, v_{N\backslash\{i\}}) \Big) \\ &\quad - \sum_{i\in N'} \Big(w(N \cup N', v_{N\cup N'}) - w(N \cup N' \backslash \{i\}, v_{N\cup N'\backslash\{i\}}) \Big) && \text{(by (3))} \\ &\geq w(N, v_N) - \sum_{i\in N\backslash N''} \Big(w(N, v_N) - w(N \backslash \{i\}, v_{N\backslash\{i\}}) \Big). && \text{(by (4))} \end{aligned}$$

Thus, by Lemma 1, we get

$$\sum_{i\in N''} u_i(f_i(N \cup N', v_{N\cup N'}); v_i) + u_0(f(N \cup N', v_{N\cup N'})) \geq \sum_{i\in N''} u_i(f_i(N, v_N); v_i) + u_0(f(N, v_N)),$$

as desired. ■

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