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New Approach to Estimating the Productivity of Public Capital: Evidence from 22 OECD Countries

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Abstract

Investigating the productivity of public capital is a long-standing issue in one strand of macroeconomic literature. This study develops a new approach to estimate the output elasticity of public capital using a vector autoregressive (VAR) model with identification restrictions derived from a theoretical model. Our empirical analysis of 22 OECD countries for the period 1960–2019 reveals that public capital accumulation has a positive effect on GDP in both the short- and long-run horizons in all countries, supporting both demand-stimulating and growth-enhancing effects. Furthermore, the estimated output elasticity of public capital lies within a reasonable range, between 0 and 0.5, and, as in the literature, shows substantial differences across countries. Therefore, the proposed methodology is valid for studying public capital productivity.

Keywords: Public capital, Hierarchical panel VAR model, Max share identification.

JEL classification: E62, H54, C32, C33.

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1 Introduction

Understanding public capital productivity has been a long standing issue in macroeconomics since the pioneering work of Aschauer (1989).[1] For instance, 578 estimates of the output elasticity of public capital in 68 studies are collected for a meta-analysis by Bom and Ligthart (2013).[2] However, existing empirical studies report a wide range of estimates across regions, sample periods, and specifications, and no consensus has been reached despite considerable research efforts. In addition, the analysis becomes much more complicated because public capital accumulation has two different effects on the economy: a short-run demand-stimulating effect and a long-run growth-enhancing effect. Public investment for public capital accumulation plays an important role in fiscal stimulus packages to cope with short-run recessions, whereas public capital itself contributes to long-run economic growth as an essential factor in the production function. Therefore, researchers addressing this issue should ideally aim to estimate both (i) the short- and (ii) long-run effects of public capital on output, and (iii) the output elasticity of public capital in a widely accepted framework. To the best of our knowledge, no previous studies have provided these three estimates combined.

This study proposes a novel method to derive estimates from a unified empirical framework. The key ingredient of this study, in terms of distinguishing it from existing studies, is the use of theoretical implications for identifying structural shocks and estimating the output elasticity of public capital simultaneously. In other words, our proposed method imposes identification restrictions extracted from a dynamic stochastic general equilibrium (DSGE) model on structural vector autoregressive (VAR) model. Specifically, we construct a neoclassical macroeconomic model with public capital accumulation *à la* Baxter and King (1993) for extracting the restrictions which can separate public investment shocks of our interest from other shocks by focusing on the long-run effects of the shocks in the model (i.e., the changes in steady-state values to permanent shocks).

[1]The first study to estimate public capital productivity is Mera (1973) on the Japanese economy.

[2]Núñez-Serrano and Velázquez (2017) conducts an even broader meta-analysis, covering about 2,000 estimated elasticity from 145 papers.

Focusing on the steady-state effects of shocks enables our analysis to be accomplished within a parsimonious model without price stickiness.

The proposed model-based identification method has two main advantages. First, our analysis can be completed using a minimal bivariate VAR model comprising only the public capital-to-GDP ratio and public capital. This parsimoniousness of the VAR system is crucial because public capital is recorded annually, which then offers only low data frequency and small sample sizes. Without a model-based identification, these problems make it difficult to identify public investment shocks that involve public capital accumulation. Indeed, the identification assumptions based on the simultaneous exogeneity of fiscal variables on the grounds of implementation lag, proposed by Blanchard and Perotti (2002), is no longer plausible using low-frequency data, and the sign restrictions proposed by Mountford and Uhlig (2009) also suffer from the degree-of-freedom problem when several variables are included to impose restrictions in the small samples. The identification strategy presented in this study is resilient to the difficulties stemming from the properties of the data. Second, we compute the output elasticity of public capital consistent with the production function using the impulse responses (IRs) obtained from VAR model. The output elasticity of public capital can be solved analytically from the solution of the theoretical model so that the elasticity can be calculated from the empirical IRs as long as some structural parameters in the model are calibrated. This study is the first to estimate the value of the output elasticity of public capital based on a VAR-based analysis.

It should also be stressed how this study imposes restrictions on the VAR model. The structural shocks in this study are characterized by the long-run effects of public investment shocks on the public capital-to-output ratio as predicted by the theoretical model. This type of restriction is normally imposed using the long-run identification method proposed by Blanchard and Quah (1989), but Faust and Leeper (1997) and Francis et al. (2014) question the preciseness of the long-run restriction at finite small sample. This also seems to be the case in our analysis, particularly because of the small sample size, as discussed above. Hence, we rely on the Max Share (MS) identification approach, recently proposed by Barsky and Sims (2011) and Francis et al. (2014) instead of the long-run identification method. In this study, public investment shocks are identified as shocks with the maximum share of forecast error variance in the public capital-to-output ratio at a finite horizon rather than those with a long-run impact at an infinite horizon. We believe that the MS approach is robust against model misspecifications when restrictions derived from the theoretical model are imposed on the VAR model, as in this study. Namely, the long-run zero restrictions might be too strong, considering the gap that would exist between the theoretical model and the real economy. To mitigate the problem of misspecification, the MS approach is a desirable method that can fulfill identification without relying too much on the consequences of a specific model.

Our analysis covers 22 OECD countries for the period from 1960 to 2019. We use public capital and GDP data released by the International Monetary Fund (IMF), which provides a comprehensive dataset constructed in the same manner as Kamps (2006). Our methodological contribution is the adoption of a hierarchical panel VAR model estimated using Bayesian technique. In a hierarchical model, the country-specific coefficients are assumed to have a common prior distribution, and the hyperparameters in the common prior are estimated using information from country-specific coefficients. This mutual use of information allows for a more accurate estimation of both country-specific coefficients and hyperparameters of the prior distribution, even for a short sample size. Indeed, the empirical results demonstrate that the Bayesian credible intervals associated with the IRs derived from the hierarchical VAR model are fairly narrow compared with those derived from the individual VAR model without the hierarchical structure.

The literature on the macroeconomic effects of public capital can be broadly categorized into two groups: the production function approach and the VAR approach. Aschauer (1989), a seminal work using the production function approach, estimates a production function with public capital and reports an elasticity as high as 0.38%. Since Aschauer (1989), many studies (e.g., Shioji, 2001; Kamps, 2006; Calderón et al., 2014) address estimation methods and document heterogeneous elasticities across countries, types of public capital, and sample periods.[3] Although the production function approach has the advantage of directly estimating the output elasticity of public capital, it also has drawbacks, such as the strong presumption of causal relationships among the variables, as pointed out in Kamps (2005). In addition, the elasticity obtained from the production function approach is limited to the long run. Thus, the short-run stimulus effects of public investment are outside the scope of this analysis. On the other hand, VAR analysis, as adopted in Kamps (2005) and De Jong et al. (2018), can overcome these disadvantages because VAR estimation is carried out without depending on prior presumptions regarding the causal relationship. However, it is no longer possible to directly estimate the value of elasticity.[4] More importantly, the estimated IRs reported in these studies often entail a relatively wide confidence band, leading to statistically insignificant results. This is because, as discussed above, VAR analysis generally contains many endogenous variables, despite the small sample size of public capital. Indeed, some IRs reported in Kamps (2005) and De Jong et al. (2018) become statistically insignificant, even when evaluating relatively narrow 68% confidence intervals. Our contribution lies in providing a tractable framework to comprehensively resolve the difficulties in both the production function approach and VAR analysis. By doing so, we show solid estimates for the short- and long-run effects of public capital on the output and output elasticity of public capital.

The findings of this study are summarized as follows. First, the IRs analysis reveals a positive effect of the public investment shock for public capital accumulation on GDP in the short and long run for all OECD countries. This finding supports both the demand-stimulus and growth-enhancing effects of public capital accumulation. Second, we estimate the output elasticity of public capital with substantial heterogeneity across

[3] A detailed survey can be found in Bom and Ligthart (2013) and Núñez-Serrano and Velázquez (2017).

[4] Kamps (2005) reports the long-run "elasticity" of GDP with respect to public capital computed from the IRs, which literally measures the percentage change in GDP per one percent change in public capital. This estimate is a useful measure to gauge the productivity of public capital, but it differs from the one documented in the production function approach literature in a strict sense. This study shows the elasticity of public capital in accordance with the production function approach using the VAR analysis results.

countries in the range of 0 – 0.5. This dispersion across estimates is consistent with the findings in the literature. An additional important aspect of this result is that our newly proposed approach works well in providing reasonable estimates of the productivity of public capital, without imposing any restrictions on signs and magnitudes. Finally, our sensitivity analyses reveal the robustness of the main results with respect to the horizon for which the MS identification restrictions are imposed. Moreover, consistent with the literature, our subsample analysis shows a significant decline in public capital productivity. As such, our approach can reasonably estimate the short- and long-run effects of public capital accumulation and its elasticity.

The remainder of this paper are organized as follows. Section 2 develops the theoretical model to extract restrictions for identifying structural shocks. Furthermore, an analytical solution for the output elasticity of public capital is derived from the theoretical model. Section 3 explains our estimation method. Specifically, the structure of the hierarchical panel VAR model, MS identification approach, and Bayesian MCMC method are described. Section 4 presents the results for the IRs and estimated elasticity of public capital for individual countries. The sensitivity analyses constructed by altering the specifications and sample periods are also presented in Section 4. Finally, Section 5 concludes the paper.

2 Identification strategy

The identification restrictions imposed on our VAR model are extracted from the steady-state features of the DSGE model. We build a neoclassical model with public capital accumulation, originally proposed by Baxter and King (1993), to introduce both permanent public investment and technology shocks. As these permanent shocks might have changed the steady-state values of the variables in the model, we search for specific variables whose steady-state values change (or do not change) in response to each permanent shock. Then, we impose the long-run implications obtained from the theoretical model on the VAR model by applying the MS identification developed by Barsky and Sims (2011) and Francis et al. (2014).

2.1 Theoretical model

The representative household receives utility from consumption c_t and disutility from labor supply l_t. The household chooses c_t and l_t to maximize the lifetime utility function, given by,

$$U = E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left[\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{l_t^{1+\varphi}}{1+\varphi}\right] \tag{1}$$

subject to the budget constraints and capital accumulation equation

$$c_t + i_t = w_t l_t + r_t^k k_{t-1} - \tau_t, \tag{2}$$

$$k_t = (1-\delta)k_{t-1} + i_t \tag{3}$$

where i_t, w_t, r_t^k, k_t and τ_t denote investment, real wages, the real rental rate, private capital stock, and lump-sum tax, respectively. Parameters ρ, γ, φ and δ are the rate of time preference, risk aversion, the inverse of the Frisch labor elasticity, and the depreciation rate, respectively. Solving the utility maximization problem yields the Euler equation and intra-temporal optimal conditions as follows:

$$c_t^{-\gamma} = E_t \frac{1 + r_{t+1}^k - \delta}{1+\rho} c_{t+1}^{-\gamma}, \tag{4}$$

$$l_t^{\varphi} = w_t c_t^{-\gamma}. \tag{5}$$

The firm in a competitive goods market produces output y_t by using the Cobb-Douglas production function

$$y_t = e^{z_t} k_{t-1}^{\alpha} l_t^{1-\alpha} k_{g,t-1}^{\alpha_g}, \tag{6}$$

where z_t is the technology level, and $k_{g,t}$ is the public capital accumulated by public investment. The parameters α and α_g indicate capital share and the output elasticity of public capital, respectively. We assume that the technology follows a random walk process with exogenous innovation according to

$$z_t = z_{t-1} + \varepsilon_t^z, \tag{7}$$

where ε_t^z denotes technology shock. Private capital and labor inputs are also assumed to be provided in competitive factor markets, resulting in factor prices given by

$$r_t^k = \alpha e^{z_t} k_{t-1}^{\alpha-1} l_t^{1-\alpha} k_{g,t-1}^{\alpha_g}, \tag{8}$$

$$w_t = (1-\alpha) e^{z_t} k_{t-1}^{\alpha} l_t^{-\alpha} k_{g,t-1}^{\alpha_g}. \tag{9}$$

In each period, the government spends g_t financed by a lump sum tax (i.e., a balanced budget). Here, we assume that government spending is allocated to public investment to accumulate public capital, leading to the accumulation equation for public capital:

$$k_{g,t} = (1-\delta_g) k_{g,t-1} + g_t, \tag{10}$$

where δ_g denotes a depreciation rate of public capital.[5] Government spending is assumed to be the θ_t fraction of the output, specified as

$$g_t = \theta_t y_t, \tag{11}$$

where the process of θ_t is given by

$$\theta_t = \theta_{t-1} + \varepsilon_t^g, \tag{12}$$

with a public investment shock ε_t^g. Finally, the economy is closed by a good market

[5]Here, the depreciation rate of public capital is defined differently from that of private capital. However, as shown below, the depreciation rate of public capital is irrelevant for our identification restriction.

clearing condition, given by

$$y_t = c_t + i_t + g_t. \tag{13}$$

The equilibrium allocation in this economy can be characterized by the stochastic sequence of $\{c_t, l_t, k_t, k_{g,t}\}_{t=0}^{\infty}$, which satisfies

$$c_t^{-\gamma} = E_t \frac{1 + \alpha e^{z_{t+1}} k_t^{\alpha-1} l_{t+1}^{1-\alpha} k_{g,t}^{\alpha_g} - \delta}{1+\rho} c_{t+1}^{-\gamma} \tag{14}$$

$$l_t^{\varphi} = (1-\alpha) e^{z_t} k_{t-1}^{\alpha} l_t^{-\alpha} k_{g,t-1}^{\alpha_g} \tag{15}$$

$$k_t = (1-\delta) k_{t-1} + (1-\theta_t) e^{z_t} k_{t-1}^{\alpha} l_t^{1-\alpha} k_{g,t-1}^{\alpha_g} - c_t \tag{16}$$

$$k_{g,t} = (1-\delta) k_{g,t-1} + \theta_t e^{z_t} k_{t-1}^{\alpha} l_t^{1-\alpha} k_{g,t-1}^{\alpha_g}, \tag{17}$$

given the initial values of private and public capital and the exogenous variables $\{\theta_t\}_{t=0}^{\infty}$ and $\{z_t\}_{t=0}^{\infty}$.

2.2 Identification restrictions

We focus on the steady-state values of specific variables in the model to extract the identification restrictions imposed on the VAR model because our theoretical model contains permanent shocks that can affect the steady-state values of the variables. Evaluating the equilibrium conditions, denoted by equations (14) – (17), in the steady-state values, we obtain the following equations:

$$\alpha e^{z} k^{\alpha-1} l^{1-\alpha} k_g^{\alpha_g} = \rho + \delta \tag{18}$$

$$n^{\varphi} = (1-\alpha) e^{z} k^{\alpha} l^{-\alpha} k_g^{\alpha_g} c^{-\gamma} \tag{19}$$

$$\delta k = (1-\theta) e^{z} k^{\alpha} l^{1-\alpha} k_g^{\alpha_g} - c \tag{20}$$

$$\delta k_g = \theta e^{z} k^{\alpha} l^{1-\alpha} k_g^{\alpha_g} \tag{21}$$

where the letters without time subscripts represent steady-state values.

Let $\hat{x} \equiv (x^{new} - x^{old})/x^{old}$ denote the change in the steady-state value in response to permanent shocks. Note that the economy shifts from the *old* steady state to the *new* one once permanent shocks occur in our model. After algebraic manipulation, as illustrated in Appendix A, we can derive the following bivariate matrix representation:

$$\begin{bmatrix} \widehat{\left(\frac{k_g}{y}\right)} \\ \hat{k}_g \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \Psi_g & \Psi_a \end{bmatrix} \begin{bmatrix} \hat{\theta} \\ \hat{\zeta} \end{bmatrix} \tag{22}$$

where

$$\Psi_g = \frac{(1-\alpha)[\alpha\delta(\gamma+\varphi) - (\rho+\delta)\{\gamma+\varphi(1-\theta)\}]}{\{(1-\alpha)(\gamma+\varphi) - \alpha_g(1+\varphi)\}\{\alpha\delta - (1-\theta)(\rho+\delta)\}},$$

and

$$\Psi_a = \frac{1+\varphi}{(1-\alpha)(\gamma+\varphi) - \alpha_g(1+\varphi)}.$$

Equation (22) demonstrates how permanent changes in the exogenous variables affect the steady-state values of the public capital-to-output ratio and public capital, leading to feasible constraints for distinguishing between public investment and technology shocks. Specifically, we can derive from equation (22) that the theoretical implication on the long-run effects of the shocks is that both the public capital-to-output ratio and public capital are affected by a permanent shift in the share of public investment in output, while only public capital is affected by a permanent shift in technology. Hence, we consider a shock with a long-run impact on the public capital-to-output ratio a public investment shock in the following VAR analysis. We emphasize here that our analysis is conducted with the MS identification, developed by Francis et al. (2014), rather than the long-run restriction that is subject to impreciseness in finite samples criticized by Faust and Leeper (1997).[6] In practice, the public investment shock in our VAR model explains most of the forecast error variance in the public capital-to-output ratio in an arbitrage long-run horizon.

[6] In fact, the data we use has only 60 samples for the time-series dimension, so the criticism by Faust and Leeper (1997) is likely to be applicable to our data.

Furthermore, equation (22) also provides an analytical solution of α_g conditioned on the long-run responses as follows:[7]

$$\alpha_g \mid \hat{k}_g, \widehat{\left(\frac{k_g}{y}\right)} = \frac{(1-\alpha)(\gamma+\varphi)}{1+\varphi} - \frac{(1-\alpha)\left[\alpha\delta(\gamma+\varphi) - (\rho+\delta)\left\{\gamma+\varphi(1-\theta)\right\}\right]}{(1+\varphi)\left\{\alpha\delta-(1-\theta)(\rho+\delta)\right\}} \frac{\widehat{\left(\frac{k_g}{y}\right)}}{\hat{k}_g}. \tag{23}$$

Once the values of the deep parameters in the theoretical model are exogenously fixed, we can compute the productivity of public capital using the long-run impulse responses obtained from the VAR model.

3 Estimation model

We estimate a panel VAR model consisting of the public capital-to-GDP ratio and public capital for 22 OECD countries to examine the dynamic effects of public investment shocks on GDP and the productivity of public capital. Within this bivariate VAR system, we isolate public investment shocks by characterizing their long-run effects on the public capital-to-GDP ratio, which is consistent with the theoretical outcome in equation (22).

The notable features of our empirical exercises are twofold. First, we postulate a hierarchical structure of the parameters in our panel VAR system. In the hierarchical panel VAR model, the country-specific parameters in VAR model are estimated under the assumption of common prior distributions, while the hyperparameters in the common prior are also estimated using the set of country-specific parameters. In other words, both the individual country-specific parameters and common hyperparameters are estimated by exploiting mutual information, leading to more accurate estimates than the standard panel VAR model without a hierarchical structure. Second, the identification restriction that only public investment shocks have a long-run effect on the public capital-to-GDP ratio is imposed on the VAR model using the MS identification approach instead of the traditional long-run restrictions. Francis et al. (2014) demonstrate by the small-

[7] Detail derivation of the analytical solution of α_g is also found in Appendix A.

sample Monte Carlo experiments that MS identification reduces the bias in the impulse responses compared with the long-run restriction, supporting the impreciseness of the long-run restriction in a finite sample argued in Faust and Leeper (1997). As detailed later, data on public capital are available only at an annual frequency from 1960 to 2019, so the use of both techniques allows for more precise estimation in our relatively small samples.

The following describes the structure of the hierarchical panel VAR model and the implementation of MS identification. Finally, we outline a Bayesian Markov Chain Monte Carlo (MCMC) algorithm to estimate our model.

3.1 Hierarchical panel VAR model

Let us denote the vector of endogenous variables for country n ($n = 1, \cdots, N$) by $Y_{nt} = [\Delta \ln \left(k^G_{nt}/y_{nt}\right), \Delta \ln k^G_{nt}]'$. Because we are interested in the effects of a permanent shift in public investment, the variables in the VAR model are contained in log-first differentials. The VAR(p) model for country n is formulated as

$$Y_{nt} = B_{n0} + \sum_{s=1}^{p} B_{ns} Y_{nt-s} + u_{nt}, u_{nt} \sim \mathcal{N}(0, \Omega_n) \tag{24}$$

where B_{n0} and $B_{ns}, (s = 1, \cdots, p)$ represent the vectors of the constant term and VAR coefficient matrices, respectively. The vector of reduced-form innovations u_{nt} follows a multivariate normal distribution with the variance-covariance matrix Ω_n. By denoting $X_{nt} = I \otimes \left[1, Y'_{nt-1}, \cdots, Y'_{nt-p}\right]$ and $\beta_n = \left[B'_{n0}, vec(B'_{n1}), \cdots, vec(B'_{np})\right]'$, equation (24) can be transformed into a simple linear regression form:

$$Y_{nt} = X_{nt}\beta_n + u_{nt}, u_{nt} \sim \mathcal{N}(0, \Omega_n), \tag{25}$$

where I is a 2×2 identity matrix, $\otimes$ is the Kronecker product, and the *vec* operator transforms the matrix $B_{ns}, (s = 1, \cdots, p)$ into a column vector by stacking each column of B_{ns}.

We assume a linear relationship between reduced-form innovations u_{nt} and orthonormal disturbances e_{nt}, as follows:

$$A_n u_{nt} = D_n^{\frac{1}{2}} e_{nt}, e_{nt} \sim \mathcal{N}(0, I). \tag{26}$$

The elements in e_{nt} are assumed to be mutually independent and have unit variance; thus, the diagonal element in D_n, denoted by δ_n, corresponds to the variance of each orthogonal shock. Under the representation of equation (26), the variance-covariance matrix of u_{nt} is

$$\Omega_n = (A_n^{-1}) D_n (A_n^{-1})'. \tag{27}$$

Matrix A_n is estimated once assuming a lower triangular matrix with diagonal elements equal to one, meaning that the orthogonal disturbances e_{nt} are the shocks identified by recursive restriction. The identification process detailed in the next subsection extracts the structural shocks of interest $\tilde{e}_{nt}$ while maintaining the variance-covariance structure Ω_n based on the MS identification approach. Similar to β_n, we denote α_n as a stacked vector of the lower triangular elements in A_n.

Hierarchical structures are imposed on the country specific coefficients α_n and β_n, as follows:

$$\alpha_n = \bar{\alpha} + \mu_n, \mu_n \sim \mathcal{N}(0, \Sigma_\mu), \tag{28}$$

$$\beta_n = \bar{\beta} + \nu_n, \nu_n \sim \mathcal{N}(0, \Sigma_\nu), \tag{29}$$

implying that the coefficients of each country share a common prior distribution. In other words, the parameters for each country are random variables drawn from a common distribution. Based on the Bayesian technique, the conditional posterior distributions of α_n and β_n are computed from the data and prior distributions (28) and (29), respectively, and, the conditional posteriors for $\bar{\alpha}$ and $\bar{\beta}$ can be specified by regarding the individual draws of α_n and β_n as observations. This mutual use of country-specific and cross-sectional information cyclically improves the accuracy of both estimates in the hierarchical model.

3.2 Max share identification

The public investment shock in this study is distinguished from the technology shock in terms of its long-run effects on the public capital-to-GDP ratio, as demonstrated in equation (22). As stressed repeatedly, this identification restriction is imposed using the MS identification method instead of the long-run restriction. Specifically, a public investment shock is characterized as a shock that maximizes the forecast error variance share of the public capital-to-GDP ratio at a given long horizon h. Horizon h is set at 20 years in the benchmark estimation, and the robustness of our results is checked by altering h to 10, 40, and 60 years.

In practice, the MS identification proceeds as follows. First, the h-period-ahead forecast error for Y_l is described by equations (24) and (26) as

$$Y_{nt+h} - Y_{nt+h|t} = \sum_{\tau=0}^{h-1} C_{n\tau} u_{nt+h-\tau} = \sum_{\tau=0}^{h-1} C_{n\tau} A_n^{-1} D_n^{\frac{1}{2}} e_{nt+h-\tau} = \sum_{\tau=0}^{h-1} \Xi_{n\tau} e_{nt+h-\tau}, \tag{30}$$

where $Y_{nt+h|t}$ is the h-period-ahead forecast of Y_n conditional on period-t information, $C_{n\tau}$ is the VMA coefficient associated with reduced-form innovations, and the (i, j) element in $\Xi_{n\tau}$ corresponds to the impulse response of variable i to the jth orthogonal shock in e_{nt} at horizon τ.[8] This reminds us that A_n is assumed to be a lower triangular matrix such that $\Xi_{n\tau}$ is the impulse response obtained under the so-called recursive restriction. Next, the orthonormal matrix Q, such as $QQ' = I$, yields any structural shocks in any identification as $\tilde{e}_{nt+h-\tau} = Q' e_{nt+h-\tau}$ and the corresponding impulse responses as $\tilde{\Xi}_{n\tau} = \Xi_{n\tau} Q$, leading

[8] The IRs to reduced-form innovations, denoted as $C_{n\tau}, (\tau = 0, \cdots, h-1)$, can be represented as a function of the VAR coefficients, $B_{ns}, (s = 1, \cdots, p)$. Thus, the IRs to orthogonal shocks, denoted by $\Xi_{n\tau}, (\tau = 0, \cdots, h-1)$, can be derived from the estimates of B_{ns}, A_n, and D_n.

to the new representation:

$$Y_{nt+h} - Y_{nt+h|t} = \sum_{\tau=0}^{h-1} \Xi_{n\tau} Q Q' e_{nt+h-\tau} = \sum_{\tau=0}^{h-1} \tilde{\Xi}_{n\tau} \tilde{e}_{nt+h-\tau}. \tag{31}$$

Then, the forecast error variance share of the ith variable attributed to the jth shock in $\tilde{e}_{nt}$ at horizon h can be obtained using

$$\omega_{ij}\left(q(h)\right) = \frac{\iota_i' \left[\sum_{\tau=0}^{h-1} \tilde{\Xi}_{n\tau} \iota_j \iota_j' \tilde{\Xi}_{n\tau}'\right] \iota_i}{\iota_i' \left[\sum_{\tau=0}^{h-1} \tilde{\Xi}_{n\tau} \tilde{\Xi}_{n\tau}'\right] \iota_i} = \frac{\iota_i' \left[\sum_{\tau=0}^{h-1} \Xi_{n\tau} q q' \Xi_{n\tau}'\right] \iota_i}{\iota_i' \left[\sum_{\tau=0}^{h-1} \Xi_{n\tau} \Xi_{n\tau}'\right] \iota_i}, \tag{32}$$

where ι_i is a 2×1 indicator vector containing 1 in ith element and 0 otherwise, and $q = Q\iota_j$ is the jth column vector of Q, resulting in $qq' = 1$.

Finally, we solve the following maximization problem to identify the public investment shock that maximizes the forecast error variance share of the public capital-to-GDP ratio at horizon h:

$$\max_{q} \omega_{1j}\left(q(h)\right) = \frac{\iota_i' \left[\sum_{\tau=0}^{h-1} \Xi_{n\tau} q q' \Xi_{n\tau}'\right] \iota_i}{\iota_i' \left[\sum_{\tau=0}^{h-1} \Xi_{n\tau} \Xi_{n\tau}'\right] \iota_i}, \text{subject to } qq' = 1. \tag{33}$$

As Faust (1998) show, the solution q^* is given by the eigenvector associated with the maximum eigenvalue of $\sum_{\tau=0}^{h-1} \Xi_{n\tau} \Xi_{n\tau}'$, where $\Xi_{n\tau}$ is the impulse response function obtained under recursive restrictions, and is readily computed after sampling α_n, β_n and δ_n for each country. Once the optimal q^* is resolved, the impulse response vectors at horizon τ of interest can be derived as $\Xi_{n\tau} q^*$.

We assume a hierarchical structure of the impulse responses, as in Canova and Pappa (2007) and Pappa (2009). Let H and ξ_n denote the maximum horizon of impulse responses and the stacked vector of each country n's impulse responses to a public investment shock, respectively. Hence, ξ_n is a $2H \times 1$ vector that stores the responses of the public capital-to-GDP ratio and the public capital to public investment shock until horizon H in this order. Similar to equations (28) and (29), we impose a hierarchical structure on ξ_n as:

$$\xi_n = \bar{\xi} + \eta_n, \eta_n \sim \mathcal{N}(0, \Sigma_\eta), \tag{34}$$

where $\bar{\xi}$ is a country invariant prior mean in the IRs, termed the "typical" response in Pappa (2009)'s terminology. Σ_η is assumed to be a diagonal matrix in which the variance of each response is stored, and we let the variance of the responses at horizon h be $\frac{0.2}{h}$, $h = 1, \cdots, H$ exogenously. The "typical" response is estimated by exploiting the information of estimated impulse responses for each country in a similar manner to $\bar{\alpha}$ and $\bar{\beta}$.

3.3 Bayesian MCMC estimation

We estimate the VAR model by using the Gibbs sampler of the Bayesian MCMC method. The estimated parameters are country-specific parameters, denoted by α_n, β_n and δ_n, $(n = 1, \cdots, N)$, and the hyperparameters in common priors, denoted by $\bar{\alpha}$, $\bar{\beta}$, Σ_μ, and Σ_ν. Moreover, the "typical" IRs are subject to estimation by exploiting a set of information on individual impulse responses $\{\xi_n\}_{n=1}^N$.

By letting $Z_n = \{Y_{nt}, X_{nt}\}_{t=1}^T$, the MCMC algorithm we apply is summarized as follows:

1. Set initial values for α_n, β_n, δ_n, $\bar{\alpha}$, $\bar{\beta}$, Σ_μ, and Σ_ν.

2. Draw β_n from $\beta_n \mid \alpha_n, \delta_n, \bar{\beta}, \Sigma_\nu, Z_n$ for each country $n = 1, \cdots, N$.

3. Draw α_n from $\alpha_n \mid \beta_n, \delta_n, \bar{\alpha}, \Sigma_\mu, Z_n$ for each country $n = 1, \cdots, N$.

4. Draw δ_n from $\delta_n \mid \beta_n, \alpha_n, Z_n$ for each country $n = 1, \cdots, N$.

5. Compute ξ_n using α_n, β_n and δ_n based on the Max Share identification.

6. Draw $\bar{\beta}$ from $\bar{\beta} \mid \{\beta_n\}_{n=1}^N, \Sigma_\nu$.

7. Draw Σ_ν from $\Sigma_\nu \mid \{\beta_n\}_{n=1}^N$.

8. Draw $\bar{\alpha}$ from $\bar{\alpha} \mid \{\alpha_n\}_{n=1}^N, \Sigma_\mu$.

9. Draw Σ_μ from $\Sigma_\mu \mid \{\alpha_n\}_{n=1}^N$.

10. Draw $\bar{\xi}$ from $\bar{\xi} \mid \{\xi_n\}_{n=1}^N, \Sigma_\eta$.

11. Return to 2.

We can see from the above algorithm that the information on the country-specific parameters and the hyperparameters in common prior is used for mutual estimation. The posterior distributions for the country-specific parameters shown in Steps 2 and 3 are constituted conditional on the hyperparameters whereas those for the hyperparameters shown in Steps 6 and 8 are conditional on the country-specific parameters. It should be also noted that the identification process is inserted into Step 5 after sampling parameters for individual country, and the sampled impulse responses are used for generating the "typical" responses in Step 10. As is standard, the posterior distributions for both the VAR and contemporaneous parameter(s), variances, and variance-covariance matrices in our empirical model can be derived as (multivariate) normal, inverse-gamma, and inverse-Wishart distributions, respectively.

4 Empirical results

4.1 Data and specification

We collect data on public capital and GDP from "IMF Investment and Capital Stock Dataset, 2021", those of which are denoted as "*kgov_rppp*" and "*GDP_rppp*", respectively, in the dataset. Kamps (2006) originally constructed a comprehensive dataset of public capital for 22 OECD countries covering the period 1960–2001, and subsequently the IMF has updated the dataset to time-series and cross-sectional dimensions in line with the methodology by Kamps (2006) and Gupta et al. (2014). The latest dataset, published in 2021, covers 194 countries and regions and extends the end of the sample period to 2019.

However, some data especially for emerging market economies and low-income developing countries, were recorded only in recent years. Therefore, we focus on 22 OECD countries with complete data from 1960 to 2019, which also allows for a reasonable comparison with the literature.

As discussed above, the VAR model contains the public capital-to-GDP ratio and public capital in the log first-difference form. Because we use annual frequency data, the lag length in the VAR model is set to be two to coping with the degree of freedom problem. We implement the MCMC iterations for drawing each parameter 60,000 times and discard the first 10,000 draws as a burn-in. To mitigate autocorrelation among each sequential draw, only every 10-th draw in our MCMC sampling is stored; thus, using a total of 5,000 draws are used for inference. Moreover, we save only the draws in which the roots of the VAR coefficients (i.e., β_n and $\bar{\beta}$) are inside the unit circle to ensure the stationarity of the VAR system.

The prior distributions of α_n and β_n can be specified as

$$\alpha_n \sim \mathcal{N}(\bar{\alpha}, \Sigma_\nu), \ \ \beta_n \sim \mathcal{N}(\bar{\beta}, \Sigma_\mu) \tag{35}$$

from equations (28) and (29). As mentioned above, the hyperparameters in the priors are estimated using the Bayesian MCMC estimation, indicating that the parameters in priors also have prior distributions. Here, we assume diffuse priors on $\bar{\alpha}$ and $\bar{\beta}$ and adopt an inverse Wishart distribution for the priors on Σ_μ and Σ_ν.

$$\Sigma_\mu \sim \mathcal{IW}(100, 0.01I), \ \ \Sigma_\nu \sim \mathcal{IW}(100, 0.01I), \tag{36}$$

where $\mathcal{IW}$ denotes an inverse Wishart distribution. The prior for the inverse of the lth diagonal element in D_n, denoted by δ_n^l, is assumed to be a gamma distribution, given as

$$\left(\delta_n^l\right)^{-1} \sim \mathcal{G}\left(\frac{10}{2}, \frac{0.01}{2}\right) \tag{37}$$

where $\mathcal{G}$ represents the gamma distribution. Finally, the prior for "typical" IRs is set to be diffuse distribution in the same way as $\bar{\alpha}$ and $\bar{\beta}$.

4.2 Effects of public investment shock

Figure 1 shows the "typical" responses of public capital and GDP to public investment shocks, where the solid lines and shaded areas indicate the median responses and the 90% credible intervals. The GDP response computed by subtracting the responses of public capital from that of the public capital-to-GDP ratio. All the IRs below (i.e., typical and individual responses) are depicted as responses to the one standard deviation shock in the public capital-to-GDP ratio located at the top of the endogenous variables in our VAR model; therefore, the quantitative effects of public investment cannot be directly compared across countries. This can be achieved by computing the output elasticity of public capital, as described in the next section.

Figure 1 shows that a public investment shock with public capital accumulation has a positive effect on GDP, which is consistent with the general statement in the literature (e.g., Kamps, 2005) that a public investment shock has a positive effect on output. Our results demonstrate that a 1.0% increase in public capital simultaneously induces a 1.6% increase in GDP in the impact period, and a 21% increase in public capital increases GDP by 10% in the long run. In short, our analysis supports both the short-run demand-stimulus effects and the long-run growth-inducing effect of public investment shocks.

It is also worth noting that the IRs are estimated quite accurately, as narrow credible intervals are observed even when evaluating the 90% bands. Compared to similar VAR analyses such as Kamps (2005) and De Jong et al. (2018), our study successfully derives statistically significant evidence from a limited number of time-series observations. The sharp estimates of the IRs are fully due to the adoption of a hierarchical structure in the VAR model. This virtue of the hierarchical model further underscores the scrutiny of IRs in individual countries.

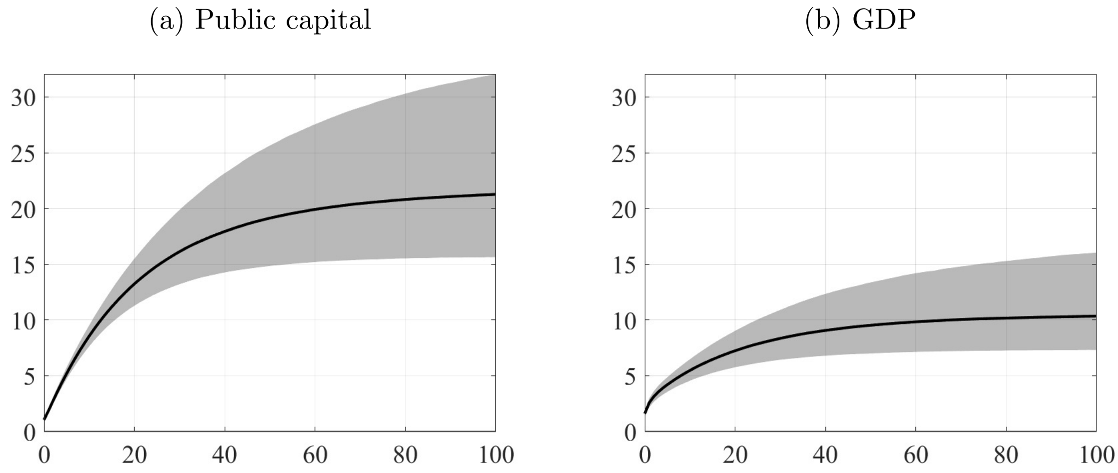

Figure 1: "Typical" impulse responses to public investment shock

Notes: The solid lines and the shaded areas correspond to the median of impulse response functions and 90% credible intervals, respectively. The horizontal axis shows the years after the shock.

Figure 2 displays the individual IRs of the GDP to public investment shocks for 22 OECD countries, where the solid lines and shaded areas indicate the median responses and 90% credible intervals, as in Figure 1. For comparison, we also plot those obtained from individual estimations without the hierarchical structure as dashed lines. A comparison between the two methods highlights the advantages of the hierarchical model over individual estimations. The IRs derived from the hierarchical model lead to the general conclusion that public investment shocks involving public capital accumulation have significantly positive effects on the GDP of all countries, which is contrary to the results obtained from individual estimations. Approximately half of the IRs in the individual estimations are, similar to Kamps (2005) and De Jong et al. (2018), statistically insignificant for all or most of the horizons, possibly because of the short sample period. Hence, the hierarchical model we adopt largely contributes to obtaining more precise estimates of country-specific responses, even in a small sample.

A possible drawback of this approach is that the shapes of the IRs may be similar in each country because of the assumption of a common prior distribution. Indeed, IRs in individual countries generally show a positive persistent response to reaching a new steady state. However, a close look at Figure 2 reveals that a noticeable difference in the shape of the IRs across the two approaches is detected only in a few countries (Iceland, Ireland, New Zealand, and the United Kingdom). In most countries, we can observe a case in which either the magnitude of the response varies without changing its sign or the credible intervals narrow without changing the median response. The representative countries for the former are Australia, Belgium, and France, whereas those for the latter are Austria, Germany, and the United States. Given these empirical findings, in addition to the relatively loose prior for individual responses in equation (34), we consider that the defects resulting from adopting the hierarchical model are limited. Rather, the responses observed particularly in Greece, Portugal, and Spain are estimated more precisely by benefiting from the hierarchical model; therefore, it can be concluded that the advantage of yielding more precise estimates is dominant in the possible disadvantage, at least in our analysis.

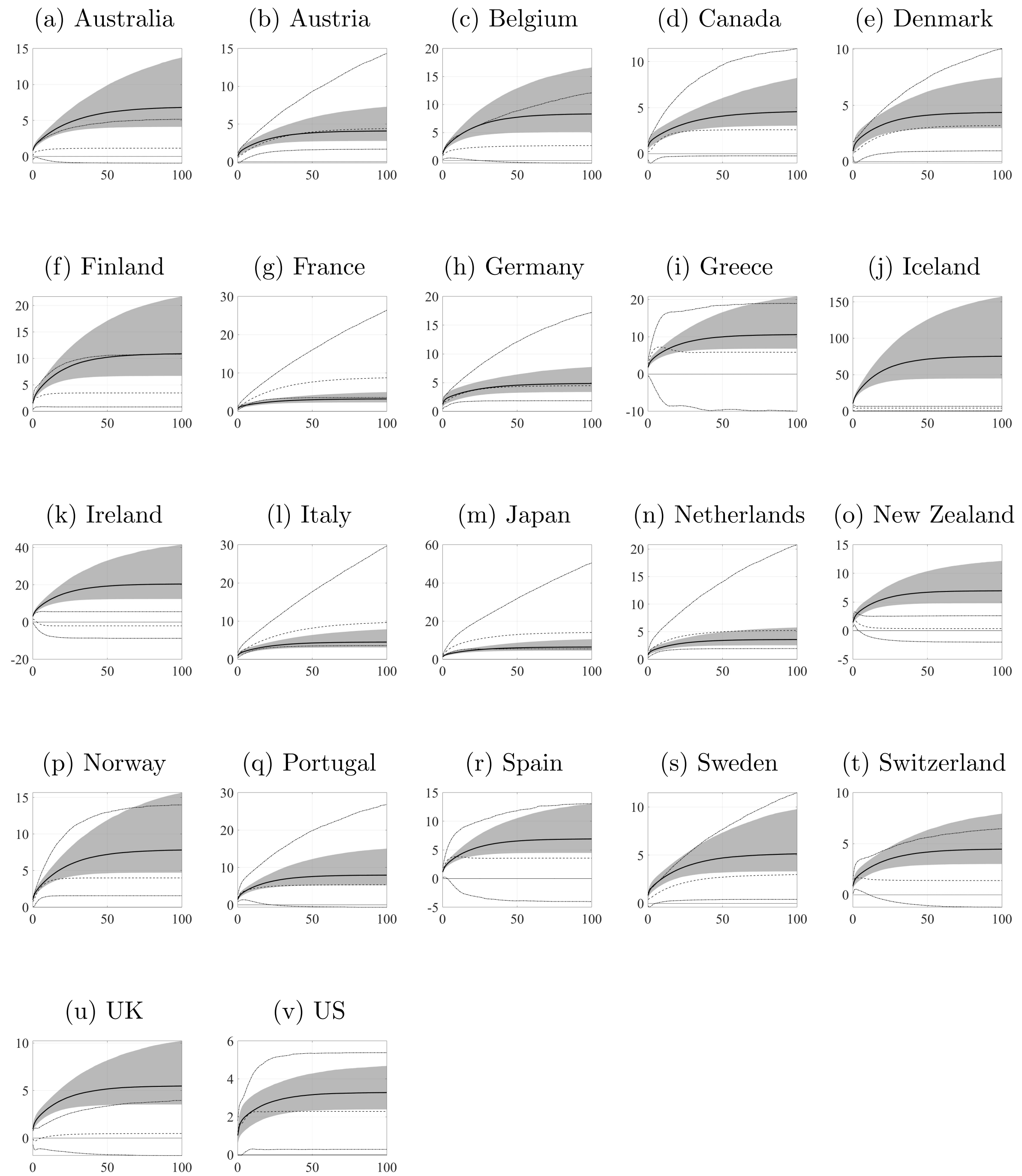

Figure 2: Individual response of GDP to public investment shock

Notes: The solid lines and the shaded areas correspond to the median of impulse response functions and 90% credible intervals, respectively. The horizontal axis shows the years after the shock. The dashed lines denote the median responses and 90% credible intervals obtained from individual estimation under the diffuse priors. That is, the dashed lines correspond to the estimates from the model without the hierarchical structure.

4.3 Productivity of public capital

Table 1 reports the estimates of the output elasticity of public capital computed using the IRs and model parameters based on equation (23), with the median in the top row and the 5th and 95th percentiles in parentheses of the bottom row. The IRs are obtained from the previous section and the parameter values required for computing them are calibrated as follows. First, the public investment-to-GDP ratio, denoted by θ, is computed using data from the IMF dataset used in the main VAR analysis. Using a time series of government investment (labelled as "*igov_rppp*") provided in the dataset, we compute the public-investment-to-GDP ratio simply by dividing "*igov_rppp*" by "*GDP_rppp*". Second, we collect the corresponding items for the capital share and depreciation rate, denoted by α and δ respectively, from "Penn World Table, version 10.01". Note that the Penn World Table provides "Share of labour compensation in GDP at current national prices", so that the capital share is constructed by subtracting the labor share from one. Finally, the remaining parameters are set exogenously to values frequently used in the literature: the rate of time preference (ρ) of 0.05, risk aversion (γ) of 1, and the inverse of the Frisch labor elasticity (φ) of 1. The sample averages for the period from 1960 to 2019 are used for the values of α, δ and θ in the actual computation on the elasticity. For instance, in the "typical" case, the output elasticity of public capital is calculated under parameter values of $\alpha = 0.39$, $\delta = 0.04$ and $\theta = 0.04$, which are computed by taking averages in both time-series and cross-sectional dimensions. The estimates of the output elasticity of public capital allow a direct comparison of the productivity effects of public capital across countries.

A first glance at Table 1 reveals a substantial dispersion in productivity across countries, from 0.24 in Greece and Iceland to 0.45 in the United States, in accordance with the evidence documented in the literature. Regarding individual estimates, our estimate of 0.45 for the U.S., is close to the estimate of 0.39 by Aschauer (1989), covering Aschauer (1989)'s estimate in terms of credible intervals. Overall, however, our estimates seem to be slightly higher than those Bom and Ligthart (2013) calculates based on a meta-analysis.

It would be difficult to simply compare them because the estimates can alter depends on specification, sample period and other factors, but our elasticity derived from "typical" responses which correspond to some kind of average values is 0.29, while its counterparts in Bom and Ligthart (2013) is 0.122 (All Public Capital, National, and Long-run in their Table 4). This difference is first attributed to methodological differences, for which we propose a new model-based framework that uses a method that reports slightly higher estimates. Another possible reason for this difference is the sample period. Previous studies have often pointed out a declining in productivity of public capital over time (e.g., Jong-A-Pin and de Haan, 2008), and the studies in Bom and Ligthart (2013)'s meta-analysis, which report extreme low elasticities, employ the specific sample period (e.g., 0.045 in Garcia-Milà and McGuire, 1992 for 1969–1983 in the United States; 0.054 in Holtz-Eakin and Schwartz, 1996 for 1969–1986 in the United Stats; 0.083 in Cadot et al., 2006 for 1985–1992 in France). In the sensitivity analysis below, we reexamine the elasticities by splitting the sample period into two parts to check for a decline in the productivity of public capital.

Instead, we emphasize the range of our estimates. All estimates lie within reasonable ranges, approximately between 0 and 0.5, which are frequently documented in the literature, despite not imposing any range restriction in advance. Therefore, it is reasonable to suppose that the proposed method works well and yields the output elasticity of public capital from VAR system. Our primary purpose of estimating the short- and long-run effects of public capital accumulation as well as the output elasticity of public capital can be said to be fully achieved by deriving reasonable estimates of elasticity.

Table 1: Estimates of the output elasticity of public capital: 1960-2019

AUS	AUT	BEL	CAN	DNK	FIN
0.29	0.32	0.28	0.36	0.34	0.29
(0.24 0.35)	(0.25 0.39)	(0.22 0.34)	(0.28 0.46)	(0.26 0.45)	(0.23 0.35)

FRA	DEU	GRC	ISL	IRL	ITA
0.37	0.42	0.24	0.28	0.24	0.30
(0.29 0.48)	(0.31 0.55)	(0.19 0.30)	(0.22 0.33)	(0.19 0.28)	(0.24 0.39)

JPN	NLD	NZL	NOR	PRT	ESP
0.32	0.38	0.30	0.25	0.31	0.29
(0.25 0.40)	(0.29 0.49)	(0.24 0.38)	(0.20 0.30)	(0.25 0.38)	(0.23 0.35)

SWE	CHE	GBR	USA	Typical	
0.29	0.36	0.28	0.45	0.29	
(0.22 0.36)	(0.28 0.46)	(0.22 0.35)	(0.32 0.62)	(0.24 0.34)	

Notes: This table reports the estimated output elasticity of public capital derived from the IRs based on equation (23). The value in the top row is median and the values in parentheses of the bottom row is the 5th and 95th percentiles.

4.4 Sensitivity analysis

In this subsection, we confirm the sensitivity of the main results. First, we alter the horizon h on which the MS identification restrictions are imposed from $h = 20$ to $h = 10, 40, 60$. This is because the "long-run" illustrated in theory is ambiguous in the MS approach, different from the traditional long-run restrictions. Figure 3 shows the IRs of public capital and the GDP in response to public investment shocks for different values of h. These results immediately support the robustness of our main findings regarding the horizon on which the restrictions are imposed. All IRs trace similar paths and lie within the credible intervals of the benchmark results, suggesting that our results are not only valid when restrictions are imposed on a specific horizon.

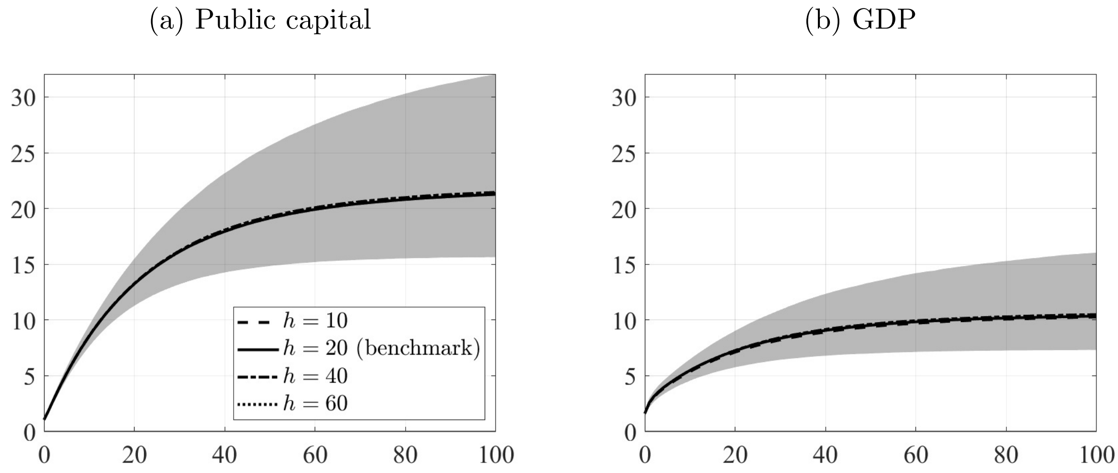

Figure 3: "Typical" impulse responses to public investment shock

Notes: This figure shows the "typical" responses of public capital and GDP to public investment shock. Solid lines and shaded areas correspond to the median and 90% credible intervals obtained from the case of $h = 20$ in the benchmark estimation. Dashed, dash-dotted and dotted lines are, respectively, the responses in the case of $h = 10$, $h = 40$ and $h = 60$.

The second sensitivity analysis examines whether the output elasticity of public capital changed according to the sample period. The sample period in our benchmark analysis is relatively long; thus, the productivity of public capital is likely to change over the sample period, as noted in the literature. In this exercise, our entire 1960–2019 sample period is divided into 1960–1999 and 1980–2019, so that the observations in both sub-samples are equal. Table 2 summarizes the estimates for the period 1960–1999 (left) and 1980–2019 (right). First, we again emphasize that our estimates are within a reasonable range in both sub samples, claiming that the validity of our method does not depend on the sample period. On top of that, all the countries including "typical" one, except for Germany, exhibits a decline in productivity of public capital in the second sub-sample compared with the one in the first sub-sample.[9] Excluding Germany, the output elasticity

[9]Puzzling behavior of the elasticity in Germany is possibly caused by German reunification. It would

of public capital decreased by approximately 42% on average. This decline in public capital productivity is consistent with that reported in the literature. Hence, our results support views widely documented in the literature, and the availability of our proposed method can be guaranteed by reconciling it with traditional results.

Table 2: Estimates of the productivity of public capital: 1960-1999 and 1980–2019

	1960–1999	1980–2019		1960–1999	1980–2019
AUS	0.28	0.14	JPN	0.28	0.16
AUT	0.28	0.16	NLD	0.33	0.26
BEL	0.25	0.15	NZL	0.29	0.13
CAN	0.37	0.17	NOR	0.23	0.12
DNK	0.30	0.17	PRT	0.29	0.15
FIN	0.26	0.16	ESP	0.25	0.13
FRA	0.33	0.24	SWE	0.25	0.15
DEU	0.37	0.49	CHE	0.32	0.18
GRC	0.21	0.11	GBR	0.25	0.14
ISL	0.24	0.13	USA	0.42	0.31
IRL	0.22	0.13	Typical	0.26	0.15
ITA	0.28	0.16			

Notes: This table reports the median estimates for the output elasticity of public capital for two sub-samples: 1960–1999 and 1980–2019.

be difficult to clarify distinctly the sources of its puzzle within our analytical framework due to the limitation of data availability and our empirical method, so we here only mention two possible hypotheses on it. One possible reason might be in the interpolation of the data for East Germany before the reunification. The imprecise estimates for East Germany data until 1990 might lead an underestimation of the elasticity during the period from 1960 to 1999. Another (positive) hypothesis is that the elasticity indeed improves because of the connection between East and West Germany. It is reasonable to suppose that economies of scale by reunification contributes to the improvement of public capital in Germany.

5 Conclusion

This study examined the effects of public capital accumulation on GDP for 22 OECD countries for the period from 1960 to 2019. The novel contribution of this study is that it provides a new approach to simultaneously estimate the short- and long-run impacts of public investment shocks and the output elasticity of public capital by imposing the restrictions obtained from the theoretical model on VAR model. Notably, we adopt a hierarchical panel VAR model with a MS identification approach to cope with the difficulty stemming from the small sample size of public capital. The main findings of our analysis are summarized as follows: First, for all the countries including "typical" responses, public investment shock involving public capital has significant positive effects on GDP in short-run and long-run horizons. Second, our estimates of the output elasticity of public capital exhibit considerable dispersion across countries, which is in line with results reported in the literature. Additionally, it should be noted that those estimates are all statistically significant at 90% credible band thanks to adopting the hierarchical structure. Finally, from sensitivity analyses, we confirm the robustness of our estimates to the horizons on which restrictions are imposed and the decline in the productivity of public capital over time, as frequently pointed out in the literature. Hence, we conclude that the proposed method contributes to the literature by providing more precise and reasonable estimates of public capital productivity.

The remaining tasks in this study are as follows. First, we should evaluate whether the effects of public capital accumulation on GDP change depending on its type. Previous studies often mentioned that core public capital such as roads and railways is more productive than others such as public buildings. The type of public capital is not distinguished in this study because we use the dataset released by the IMF in favor of the comprehensiveness of the data. Second, it is important to apply the proposed method to other emerging and developing countries. The primary purpose of this study is to propose a new approach. Therefore, we focus only on OECD 22 countries where relatively stable results are likely to be obtained because of data availability. Because the validity of our approach is confirmed in this study, we should next address the analysis of those countries.

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Appendices

A Derivations of the theoretical implication

The log-linearization of equations (18) to (21) around the old steady state yields:

$$0 = \hat{z} + (\alpha - 1)\hat{k} + (1 - \alpha)\hat{l} + \alpha_g \hat{k}_g, \tag{A.1}$$

$$\varphi\hat{l} = \hat{z} + \alpha\hat{k} - \alpha\hat{l} + \alpha_g \hat{k}_g - \gamma\hat{c}, \tag{A.2}$$

$$\hat{k} = \frac{(1-\theta)(\rho+\delta)}{\alpha\delta}\hat{y} - \frac{\theta(\rho+\delta)}{\alpha\delta}\hat{\theta} + \frac{\alpha\delta - (1-\theta)(\rho+\delta)}{\alpha\delta}\hat{c}, \tag{A.3}$$

$$\hat{k}_g = \hat{\theta} + \hat{y}, \tag{A.4}$$

$$\hat{y} = \hat{z} + \alpha\hat{k} + (1 - \alpha)\hat{l} + \alpha_g \hat{k}_g, \tag{A.5}$$

where $\hat{x} \equiv (x^{new} - x^{old})/x^{old}$ is the change in the steady-state value of the permanent shocks. We immediately obtain the first row of equation (22) as equation (A.4). Then, we can transform equation (A.2) into

$$\hat{c} = \frac{1}{\gamma}\left\{\hat{z} + \alpha\hat{k} - (\alpha + \varphi)\hat{l} + \alpha_g \hat{k}_g\right\}. \tag{A.6}$$

Subtracting equation (A.5) from equation (A.1) yields:

$$\hat{y} = \hat{k}. \tag{A.7}$$

Moreover, by substituting equation (A.7) into equations (A.5) and (A.3), we obtain

$$\hat{l} = \frac{1}{\alpha - 1}\hat{z} + \hat{k} + \frac{\alpha_g}{\alpha - 1}\hat{k}_g, \tag{A.8}$$

and,

$$\hat{k} = \hat{c} - \frac{\theta(\rho+\delta)}{\alpha\delta - (1-\theta)(\rho+\delta)}\hat{\theta}. \tag{A.9}$$

From equations (A.6) and (A.8), we can delete the labor supply, $\hat{l}$, as follows:

$$\hat{c} = \frac{1+\varphi}{(1-\alpha)\gamma}\hat{z} - \frac{\varphi}{\gamma}\hat{k} + \frac{\alpha_g(1+\varphi)}{(1-\alpha)\gamma}\hat{k}_g \tag{A.10}$$

Replacing consumption $\hat{c}$ in equation (A.9) with equation (A.10) gives

$$\hat{k} = \frac{1+\varphi}{(1-\alpha)(\gamma+\varphi)}\hat{z} + \frac{\alpha_g(1+\varphi)}{(1-\alpha)(\gamma+\varphi)}\hat{k}_g - \frac{\theta\gamma(\rho+\delta)}{(\gamma+\varphi)\left\{\alpha\delta-(1-\theta)(\rho+\delta)\right\}}\hat{\theta} \tag{A.11}$$

Finally, we obtain the second column of equation (22) from equations (A.4), (A.7), and (A.11) as follows:

$$\hat{k}_g = \frac{(1-\alpha)[\alpha\delta(\gamma+\varphi)-(\rho+\delta)\{\gamma+\varphi(1-\theta)\}]}{\{(1-\alpha)(\gamma+\varphi)-\alpha_g(1+\varphi)\}\left\{\alpha\delta-(1-\theta)(\rho+\delta)\right\}}\hat{\theta} + \frac{1+\varphi}{(1-\alpha)(\gamma+\varphi)-\alpha_g(1+\varphi)}\hat{z}. \tag{A.12}$$

Furthermore, solving equation (A.12) with respect to α_g conditioning only on $\hat{\theta}$ yields

$$\alpha_g \mid \hat{k}_g, \hat{\theta} = \frac{(1-\alpha)(\gamma+\varphi)}{1+\varphi} - \frac{(1-\alpha)\left[\alpha\delta(\gamma+\varphi)-(\rho+\delta)\left\{\gamma+\varphi(1-\theta)\right\}\right]}{(1+\varphi)\left\{\alpha\delta-(1-\theta)(\rho+\delta)\right\}}\frac{\hat{\theta}}{\hat{k}_g}, \tag{A.13}$$

which is the analytical solution to the output elasticity of public capital described in equation (23).