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Low Interest Rates, Growth, and Sustainable Fiscal Policies

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Masaya Sakuragawa* and Yukie Sakuragawa♦

Abstract

This paper establishes a growth theory that enable us to study fiscal policies in economies of low interest rates on debt. In order to explain low interest rates, we introduce financial frictions, namely, uninsurable idiosyncratic risk in capital income and borrowing constraints faced by firms, into a standard endogenous growth model. Interest rates on debt can fall below the economic growth rate, and then the government can sustain debt by running primary deficits. Low interest rates on debt arise from the shortage in liquidity, and thus those low rates are associated with low investment and slow economic growth. The choice faced by the government is either the set of deficits and slow growth or the set of surpluses and fast growth. We show that the current Japanese economy falls into a region of liquidity shortage. We evaluate fiscal policies at aiming fiscal surpluses above zero from the perspective of our model.

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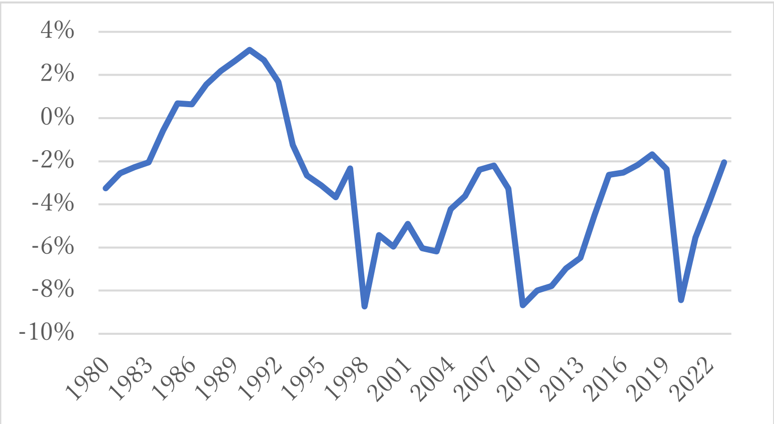
1. Introduction

The Japanese government has run primary deficits ever since 1990s (see **Figure 1A**; source, IMF), and the public debt has increased over time, but has been sustained (see **Figure 1B**; source, IMF).

In **Figure 1C**, we depict long-term rates on public bonds and GDP growth rates in real terms in Japan, showing that interest rates have been lower than growth rates, and often negative in recent years.[1]

Are low interest rates the norm or exception? While it is unusual to think of interest rates as low from the neoclassical perspective, global economic trends are supporting low interest rates on public debt. Blanchard (2019) proposes a new norm for fiscal policies. If the real interest rate facing the government is below the economic growth rate, the government could run primary deficits even when public debt reaches a significant level.

Figure 1A: Primary surplus (percent of GDP)



[1] Data sources are the Cabinet Office and the Ministry of Finance. Data are in real terms after removing the effect of inflation.

Figure 1B: General government gross debt (percent of GDP)

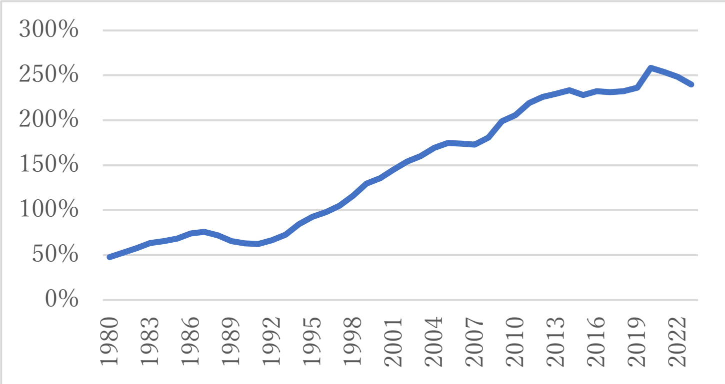
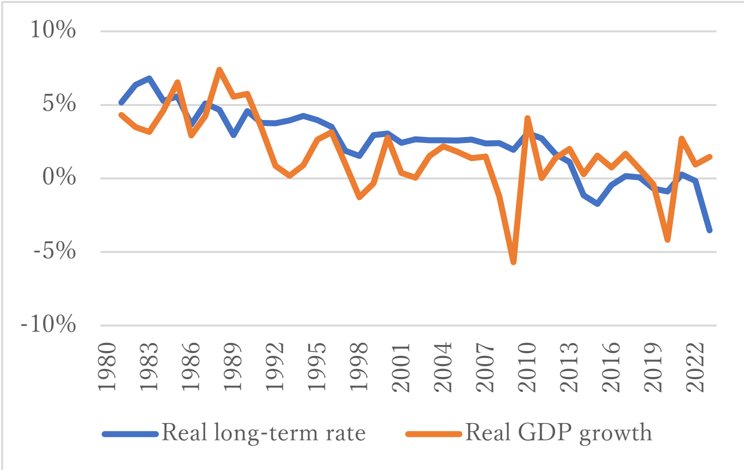


Figure 1C: Interest Rate and Growth Rate



In this paper we establish a growth theory that enable us to study fiscal policies in economies of low interest rates on debt. In order to explain low interest rates, we introduce two kinds of financial frictions, namely, uninsurable idiosyncratic risk in capital income and borrowing constraints faced by firms, into a standard endogenous growth model.

When interest rates on debt fall below the return on capital, the average return to wealth changes as the ratio of capital to public debt changes, and thus growth rates also change. Our endogenous growth model creates a channel through which interest rates are positively linked to growth rates. This channel is consistent with the natural-interest-rate view that argues in pessimism that low rates reflect the secular stagnation.[2]

Low interest rates arise from the shortage in the supply of public and private debt, namely assets to which people have easy access as a store of value. We call those assets "liquidity". (Aiyagari 1994, Aiyagari and McGrattan 1998, and Angeletos *et al.* 2023). The shortage in liquidity is associated with low investment and slow economic growth (Woodford 1990, Holmstrom and Tirole 1998 and Angeletos *et al*. 2023).

Our model provides a framework to study fiscal policies not only in the neoclassical regime but also in a regime of liquidity shortage. The neoclassical view states that governments must run primary surpluses to pay off debt, but when the economy falls into a state of liquidity shortage, interest rates on debt can fall below growth rates, and then the government can sustain debt by running deficits.

Public debt affects interest rates on debt and the economic growth rate. Fiscal policies are non-neutral to the allocation.[3] As the policy tool of eliminating the distortions of financial frictions, there is the room for further issuance of public debt so long as it is backed by taxation. By controlling public debt, the government chooses primary surpluses and at the same time the economic growth rate. A change in public debt is followed by changes in interest rates and growth rates, and determines the sustainable fiscal balance.

We examine if the Japanese economy has fallen into the "new norm", a region of liquidity shortage. We show that this is the case. The economy has shifted from an a frictionless to a region of liquidity shortage somewhere between 1995 and 2013.

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[2] Laurence Summers (2013) invoked the attention on the secular stagnation by observing the low rate of interest in the US.

[3] Public bonds are perceived as net wealth only if their value exceed the capitalized value of the stream of the future tax liabilities. Barro (1974) shows that the explanatory effects of fiscal policies hinge on if public bonds are perceived as net wealth by the private sector, and that this assumption hold when capital markets are imperfect and lives are finite.

The reasons are first the decline in the growth opportunity and secondly the secular contraction of the bank lending.

Restoring fiscal soundness has been an important policy agenda. The policy target was to set the primary surplus above zero. We evaluate this policy from the perspective of our model.

Facing low interest rates, the government would be tempted to believe that the fiscal stance is not a constraint on faster economic growth from the Keynesian point of view. This might be a case for a short time, but not be true for long. A government may be tempted to enjoy low interest rates, deficits, and fast growth, but if the government is far-sighted, it will soon realize that it is impossible to have them all.

Literature review

This paper is related to the literature that has emphasized that public debt eases financial frictions by contributing to the supply of assets that provide liquidity and/or are used as buffer stocks (Woodford 1990, Holmstrom and Tirole 1998, Farhi and Tirole 2012, and Angeletos *et al*. 2023).

This paper is related to the broad literature that studies fiscal policies in an economy at low interest rates. Blanchard (2019) proposes a new norm for fiscal policies when the real interest rate facing the government is below the economic growth rate. Ever since Aiyagari (1994), the literature has studied the optimal public debt in an economy at low interest rates (see also Aiyagari and McGrattan 1998, and Angeletos *et al*. 2023). Angeletos *et al.* (2023) studies the optimal public debt when there is the gain of debt issuance at low interest rates.

This paper is related to the literature that has argued on the fiscal sustainability in Japan. Several papers give pessimistic scenarios on fiscal sustainability from the neoclassical point of view, but their predictions have not yet explained the reality (See for example Braun and Joines 2015, Hansen and Imrohoroglu 2016, and others). Hansen and Imrohoroglu (2023) show that the unexpected fall in interest rates on debt contributed to the fiscal evidence that were different from the neoclassical predictions.

This paper is related to the broad literature that has argued if public debt crowds-in or crowds-out economic activities. Neoclassical models insist the presence of crowding-out effects on capital accumulation (see Diamond 1965, Saint-Paul 1992, and others). In contrast, Woodford (1990) shows that when interest rates are so low that public debt plays the role of liquidity, public debt can crowd in capital accumulation and economic growth. Farhi and Tirole (2012) and Hirano and Yanagawa (2017) show the similar crowding-in mechanism when rational bubbles provide liquidity.

This paper is organized as follows. In Section 2, we describe the model. In Section 3, we conduct the analysis. In Section 4, we investigate the link among interest rates, economic growth, and fiscal policies. In section 5, we study in which regime Japan stays between the neoclassical regime and the regime of liquidity shortage. In Section 6, we evaluate the Japanese fiscal policies after 2005. Section 7 concludes.

2. Model

Let us consider an economy that exists over an infinite horizon. There is a continuum of entrepreneurs who live in an infinite horizon, with the measure being unity. The agent i of type j has the preference of $E_0 \sum_{t=0}^{\infty} \beta^t \log c_{i,t}^j$, where $c_{i,t}^j$ is consumption, β is the subjective discount factor, and E_0 is the expectation operator.

At the beginning of period t, entrepreneurs make a decision on consumption and savings. Entrepreneurs are divided into two types. An entrepreneur is "active", denoted "EA", with probability q, irrespective of past history. Likewise, any entrepreneur is "inactive", denoted "EI", with probability $1 - q$. The types are revealed at the time after they make a decision on consumption and savings.

Once the type has been revealed, an active entrepreneur i has access to the technology $y_{i,t} = Ak_{i,t-1}$that transforms one unit of good into A units of goods across periods. The inactive entrepreneur has no access to production, and acts as an investor.

Active entrepreneurs are willing to engage in production. Assume that once output is produced, a fraction $\phi(< 1)$ of the gross capital income $(1 + A)k_{i,t}$ is pledgeable to outsiders. Thus, active entrepreneurs can issue securities whose payment is contingent on pledgeable income, which we call "bonds" or "debt". Inactive entrepreneurs buy bonds issued by active entrepreneurs. Inactive entrepreneurs have no ability of producing capital, and hold no pledgeable assets. They do not issue bonds, but are rather willing to buy bonds.

The government is subject to the flow-of-fund constraint

(1) $$S_t + D_t = (1 + r_t)D_{t-1},$$

where D_t is the real value of public bonds, S_t is the primary surplus, r_t is the return on public bonds. Note that $S_t = \tau_t C_t - Z_t$, where C_t is aggregate consumption, Z_t is the government expenditure, and τ_t is the consumption tax rate (a negative tax rate implies a transfer). The government does not hold assets, and unlike the private sector, can issue bonds without the guarantee of pledgeable assets so long as bonds are solvents according to (1).

3. Analysis

Behavior of entrepreneurs

At the beginning of any period, the levels of wealth are different across entrepreneurs, depending on the history of type. When an entrepreneur i was a type h ($h = EA$ or EI) last period and is a type j ($j = EA$ or EI) this period, the flow-of-funds constraint at the beginning of period t is represented as

$$w_{i,t}^{j} = \left(1 + R_t^h\right) w_{i,t-1}^{h} - (1 + \tau_t) c_{i,t}^{j},$$

where R_t^h denotes the return for the agent of type h ($h = EA, EI$).

An active entrepreneur i who retains own wealth $w_{i,t}^{EA}$ raises funds by issuing bonds $b_{i,t}$, to finance $k_{i,t}$:

(2) $$w_{i,t}^{EA} + b_{i,t} = k_{i,t} \ .$$

Likewise, an inactive entrepreneur i retains own wealth w_{it}^{EI} purchase private bonds $b_{i,t}^{EI}$ and public bonds $d_{i,t}^{EI}$:

(3) $$w_{i,t}^{EI} = b_{i,t}^{EI} + d_{i,t}^{EI}.$$

The inactive neither issue bonds because they do not hold assets pledgeable to outsiders.

Equations (2) and (3) show that the active use capital, but the inactive do not need capital. The existing capital is reallocated from inactive to active entrepreneurs, that is, the active buy capital from the inactive.

It is easy to see that the price of the existing capital is unity. The inactive who changed their type from active this period would like to sell their capital, but if the price is less than unity, they choose to consume rather than sell capital. The active decide whether to produce capital by own technology or purchase capital from others, but if the price is above unity, they choose to produce capital rather than buy.

We turn to the production side of entrepreneurs. An active entrepreneur employs the production technology if $r_{t+1} \leq A$ holds. This inequality is referred to as the *profitability constraint*. To finance larger $k_{i,t}$, he or she is willing to raise funds by issuing bonds. However, because the amount raised by issuing bonds is limited to the pledged income $\phi(1+A)k_{i,t}/(1+r_{t+1})$, this active entrepreneur is subject to the *borrowing constraint*, $(1+r_{t+1})b_{i,t} \leq \phi(1+A)k_{i,t}$. The equilibrium requires either the profitability constraint or the borrowing constraint to be satisfied with equality. The profitability constraint is satisfied with equality when the borrowing constraint does not bind with equality, otherwise, the borrowing constraint binds with equality:

$$(1+r_{t+1})b_{i,t} = \phi(1+A)k_{i,t}, \quad (4)$$

where $b_{it} = k_{i,t} - w_{i,t}^{EA}$. This expression reveals that taking the net worth $w_{i,t}^{EA}$ as given, the amount of bonds b_{it} and capital $k_{i,t}$ are determined. Furthermore, it also reveals that the pledged fraction ϕ of the total return on capital $(1+A)k_{i,t}$ accrues to the bond holders, and simultaneously implies that the unpledgeable fraction $(1-\phi)$ of the return accrues to the entrepreneur as an equity holder.

The analysis here focuses on the case when borrowing constraint binds with equality. The current profit of the active firm is

$$(1+A)k_{i,t} - (1+r_{t+1})b_{i,t} = (1-\phi)(1+A)k_{i,t},$$

where the equality uses (4). Here ROE, denoted r_{t+1}^{E}, must satisfy

$$(1+r_{t+1}^{E})w_{i,t}^{EA} = (1-\phi)(1+A)k_{i,t}. \quad (5)$$

We turn to the inactive entrepreneur. The current profit of the inactive firm is $(1+r_{t+1})\left(b_{i,t}^{EI} + d_{i,t}^{EI}\right)$. The inactive entrepreneur receives $(1+r_{t+1})w_{i,t}^{EI}$, using (3).

We now investigate returns on several assets. ROE is represented as the return on capital accruing to the active entrepreneur $(1-\phi)(1+A)$ multiplied by the leverage $1/\{1-\phi(1+A)/(1+r_{t+1})\}$, such that

(6) $$1 + r_{t+1}^{E} = \frac{(1-\phi)(1+A)}{1-\phi(1+A)/(1+r_{t+1})}.$$[4]

Notably, ROE is high if interest rates are low. In principle, three returns are related as follows.

(7) $$1 + A = \frac{w_{i,t}^{EA}}{k_{i,t}}(1 + r_{t+1}^{E}) + \frac{b_{i,t}}{k_{i,t}}(1 + r_{t+1}).$$

The (gross) return on capital of the LHS is a wealth-share weighted average of ROE and interest rates on debt. When the borrowing constraint binds with equality, ROE is higher and interest rates are lower than the return on capital, such that $r_t < A < r_t^E$. As interest rates rise, the equity premium $(r_t^E - r_t)$ decreases. When the borrowing constraint ceases to bind, the three returns are equal, such that $A = r_t = r_t^E$ and the equity premium becomes zero.

Entrepreneurs receive wealth that is linear in own assets, but their return on wealth is stochastic. The return for an active entrepreneur equals ROE, namely, $R_t^{EA} = r_t^E$, and the return for an inactive entrepreneur equals interest rates on debt, namely, $R_t^{EI} = r_t$.

Entrepreneurs face idiosyncratic risk on the return on investment at the stage of making the consumption/savings decision. This stochastic problem for the choice of consumption and savings is similar to the classic problem studied by Samuelson (1969) and Merton (1969). When preferences are homothetic and wealth is linear in own assets, the saving rate out of wealth is independent of the level of wealth, and in general dependent on the expected returns on investment. In principle, the effect of a change in the return on the savings rate depends on the opposing wealth and substitution effects. These two effects are completely offset when the utility is logarithmic and thus the savings rate is constant. See **Appendix A** for the derivation.

The fraction $(1 - \beta)$ of wealth $(1 + R_t^h)w_{i,t-1}^h$ is used for consumption,

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[4] Equation (6) is derived when (5) is combined with (2) and (4).

$$(1+\tau_t)c_{i,t}^{j} = (1-\beta)\left(1+R_t^h\right)w_{i,t-1}^{h},$$

and the remaining fraction β is allocated to the holding of wealth,

$$w_{i,t}^{j} = \beta\left(1+R_t^h\right)w_{i,t-1}^{h},$$

where R_t^h is stochastic, such that $R_t^{EA} = r_t^E$, and $R_t^{EI} = r_t$.

The wealth of each individual entrepreneur is different, but aggregate wealth is deterministic due to the law of large numbers. We define the aggregate variable as $X_t = \sum x_{i,t}$.The aggregate wealth of active entrepreneurs comprises the proportion q of the total wealth of active and inactive at the last period;

(8) $$W_t^{EA} = \beta q\{(1+r_t^E)W_{t-1}^{EA} + (1+r_t)W_{t-1}^{EI}\}.$$

Likewise, the aggregate wealth of inactive entrepreneurs comprises the proportion $1-q$ of the total wealth of active and inactive at the last period;

(9) $$W_t^{EI} = \beta(1-q)\{(1+r_t^E)W_{t-1}^{EA} + (1+r_t)W_{t-1}^{EI}\}.$$

By summing up the wealth of active and inactive, the aggregate wealth of entrepreneurs, denoted $W_t^E (\equiv W_t^{EA} + W_t^{EI})$, is

(10) $$W_t^E = \beta\{(1+r_t^E)W_{t-1}^{EA} + (1+r_t)W_{t-1}^{EI}\}.$$

Dividing both sides of (10) by W_{t-1}^E, and using $\frac{W_{t-1}^{EA}}{W_{t-1}^E} = q$ and $\frac{W_{t-1}^{EI}}{W_{t-1}^E} = 1-q$, the aggregate wealth of entrepreneurs grows according to

(11) $$\frac{W_t^E}{W_{t-1}^E} = \beta\{q(1+r_t^E) + (1-q)(1+r_t)\},$$

which means that wealth grows at a rate proportional to the return of a weighted average of ROE and interest rates on debt.

The ratio of the wealth of active to the wealth of inactive is immediate from (8) and (9):

(12) $$\frac{W_t^{EA}}{W_t^{EI}} = \frac{q}{1-q},$$

The wealth W_t^{EA} is the wealth of insiders of active firms, and W_t^{EI} is the wealth of outsiders of active firms and of the government. Equity is a claim of insiders of active firms and debt is a claim of the outsiders. Thus q has an interpretation of "the ratio of equity to debt". A high q means that a large number of entrepreneurs access the capital production and the ex-ante average return on capital is $q(1+A)$. Thus the ratio of equity to debt is positively related to the technology parameter q. [5]

Aggregate Wealth, Capital, and Public Debt

We next combine capital and public debt with wealth. The aggregation of (2) is described as

(13) $$W_t^{EA} + B_t = K_t.$$

Likewise, the aggregation of (3) is described as

(14) $$W_t^{EI} = B_t^{EI} + D_t^{EI}.$$

We define "liquidity" as private and public debt, namely, assets to which people have easy access as a store of value. The value of liquidity at maturity is $(1+r_{t+1})(B_t^{EI} + D_t^{EI})$, which is low if interest rates are low.[6]

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[5] We comment on the relation between our model and much debate about the huge retained earnings held by Japan firms. In this model, only the wealth of the active is directly used to finance investment, and $1-q$ will capture a measure of the so-called "problematic" retained earnings. Strictly, part of the wealth of the inactive is used for financing investment indirectly through bond holding, and thus all of the wealth of the inactive is not a problem.

[6] If debt is defined as discount bonds, equation (14) is replaced as $W_t^{EI} = p_t(B_t^{EI} + D_t^{EI})$, where p_t is the price of bonds, and become unity at maturity (period t+1). The value of liquidity at maturity is W_t^{EI}/p_t . If $1 + r_{t+1} = 1/p_t$, either specification yields the same result.

Clearing the market for private bonds requires $B_t^{EI} = B_t$, and clearing the market for public bonds requires $D_t^{EI} = D_t$.

Summing up (13) and (14), and using the two market clearing conditions yield

(15) $$W_t^E = K_t + D_t.$$

The wealth of entrepreneurs are used to finance capital and public debt.

Equations (4) and (5) are rewritten in terms of aggregate variables as

(16) $$(1 + r_{t+1})B_t = \phi(1 + A)K_t, \text{ and}$$

(17) $$(1 + r_{t+1}^E)W_t^{EA} = (1 - \phi)(1 + A)K_t.$$

Using (14), $B_t^{EI} = B_t$, and $D_t^{EI} = D_t$, and rearranging terms, (16) is rewritten as

(18) $$(1 + r_{t+1})W_t^{EI} = \phi(1 + A)K_t + (1 + r_{t+1})D_t.$$

We use (10), (15), (17), and (18) to eliminate wealth terms, and eventually derive

(19) $$K_t + D_t = \beta\{(1 + A)K_{t-1} + (1 + r_t)D_{t-1}\}.$$[7]

Let $1 + g_t (= K_t/K_{t-1})$ denote the gross growth rate of capital and $d_t (= D_t/K_t)$ denote the ratio of public bonds to capital, which is a key variable throughout this paper. Dividing both sides of (19) by K_{t-1} and rearranging terms leads to

(20) $$1 + g_t = \beta\left\{\frac{1}{1+d_t}(1 + A) + \frac{d_{t-1}}{1+d_t}(1 + r_t)\right\}.$$

The RHS represents the average return on wealth (multiplied by β), which is the sum of the return on capital $(1 + A)$ weighted by the capital share $1/(1 + d)$ and interest rates weighted by the public debt share $d/(1 + d)$. This is a variant of the

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[7] $K_t + D_t = W_t^E = \beta\{(1 + r_t^E)W_{t-1}^{EA} + (1 + r_t)W_{t-1}^{EI})\} = \beta\{(1 + A)K_{t-1} + (1 + r_t)D_{t-1}\}$, where the first equality uses (15), the second equality uses (10), the third equality uses (17) and (18).

standard representation of the AK model, such that $1+g_t=\beta(1+A)$. It is immediate that when $r_t<A$, $1+g_t$ is less than $\beta(1+A)$.

The growth rate depends negatively on the ratio of public bonds to capital d through the "portfolio effect"; a rise in d increases the public debt share $d/(1+d)$, decreases the average return on wealth, and promotes growth. The growth rate depends positively on interest rates on debt through the "liquidity effect"; a rise in the interest rate decreases the price of bonds. Agents receive the gain of liquidity from the cheaper bonds, and when the liquidity is used as a store of value, they enjoy a rise in the average return on wealth, and high growth.

This model provides an interesting implication on "secular stagnation". As is conventional, a decline in the return on capital, A, deters economic growth. A fall in interest rates also deters economic growth. When $r_t<A$, the latter provides a financial story of secular stagnation. A fall in pledgeability on debt finance or a fall in the ratio of equity to debt leads to a fall in interest rates on debt followed by growth stagnation. We find it difficult to identify the shock, either real or financial, that leads to secular stagnation.

Determinant of Interest Rates

Equation (16) shows that the tightness of the borrowing constraint is affected by the ratio of private debt to capital B_t/K_t. Using $W_t^{EI}=(1-q)W_t^E$, (14) leads to

$$(1-q)W_t^E=(1-q)(K_t+D_t)=B_t+D_t, \tag{21}$$

where the first equality uses (15). The latter equality directly leads to

$$\frac{B_t}{K_t}=1-q-qd_t$$

which means that B_t/K_t is negatively related to $d_t(=D_t/K_t)$. Public and private debt are substitutes. Combining the latter equality with (16), and rearranging terms yield

(22) $$1 + r_{t+1} = \frac{\phi(1+A)}{1-q-qd_t}.$$

A high d_t is associated with rising interest rates on debt. Taking the level of capital as given, an increase in public bonds drives investors to demand smaller private bonds that are competing assets with public bonds, which in turn leads to rising interest rates. A high ϕ is associated with rising interest rates on debt. A high ϕ means large pledgeable assets, and then entrepreneurs are able to issue large amounts of bonds. In this way, more supply of private bonds leads to rising interest rates. The supply of liquidity, whether private or public bonds, leads to rising interest rates.

Equation (22) indicates that because r is increasing in d, it implies that at some level of d, the economy shifts from a region of the binding borrowing constraint to a region of the binding profitable constraint. Substituting $r = A$ into (22) determines the threshold of d_t that separates the economy into the two regions. The following property is established.

Result 1: There exists the threshold ratio of public debt to capital, denoted d_{NF} , below which the borrowing constraint binds with equality, and above which the borrowing constraint does not bind, satisfying

($) $$d_{NF} = \max\left\{0, \frac{1-q-\phi}{q}\right\}.$$

A low ϕ means a high d_{NF}. If pledgeable assets are small, entrepreneurs are able to issue small amounts of bonds. Following (22), if ϕ is low, interest rates are low given d_t. The aggregate liquidity value at maturity, $(1 + r_{t+1})(B_t + D_t)$, tends to be low, and public bonds are permitted to be issued at large scale. Therefore, the threshold d_{NF} reflects a degree of liquidity shortage. If $d_{NF} > 0$ and $d_t < d_{NF}$, there is the shortage of liquidity, while if $d_{NF} = 0$, or $d_{NF} > 0$ and $d_t \geq d_{NF}$,, the liquidity is abundant.

The determinant of interest rates on debt is summarized as

(23) $$1 + r_{t+1} = 1 + A \text{ if } d_t \geq d_{NF}, \text{ and}$$

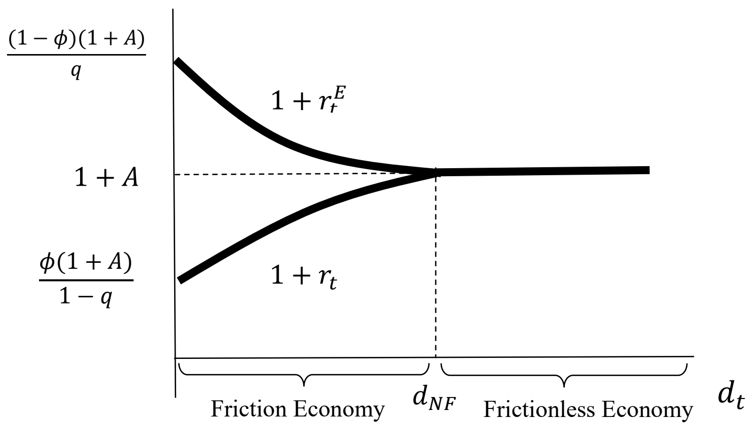
$$= \frac{\phi(1+A)}{1-q-qd_t} \text{ if } d_t < d_{NF}.$$

Once d_{NF} has been defined, we are now prepared to study the interplay between three returns. **Figure 2** illustrates how interest rates and ROE evolve as d_t changes. Interest rates are lower, and ROE is higher than the return on capital. The threshold d_{NF} divides the economy into the "frictionless economy" and the "friction economy".

When d_t is less than the threshold, ROE moves in a countercyclical way with interest rates. ROE is written as $1 + r_{t+1}^E = \frac{(1-\phi)(1+A)}{q+qd_t}$, and is decreasing in d_t, An increase in d_t leads to a decrease in the equity premium.

Given any $d_t (< d_{NF})$, the vertical difference between the two rates corresponds to the equity premium, which decreases as d_t approaches d_{NF}. Once d_t has arrived at the threshold level, interest rates on debt equal the return on capital, and the equity premium becomes zero.

Figure 2: Interest rate and ROE



Competitive Equilibrium

Let $s_t = S_t/K_{t-1}$ denote the ratio of the primary surplus to past capital. By dividing by K_{t-1}, the government's flow-of-funds constraint (1) is rewritten as

(24) $$s_t = (1 + r_t)d_{t-1} - (1 + g_t)d_t,$$

The LHS of (24) is the primary surplus necessary to cover the interest payment, while the RHS reflects the net interest payment on debt, which depends on interest and growth rates. [8]

We close the model by specifying the fiscal policy. As the policy target, the government sets the ratio of public debt to capital. When d_t is chosen, the primary surplus s_t is determined residually.

One may question if the government can control d_t because d_t includes private capital K_t as well as public debt D_t. Consider the game where the government is a leader and active entrepreneurs are followers. Entrepreneurs choose $K_t = K(D_t, r_{t+1})$ at the aggregate level by taking D_t and r_{t+1} as given, to satisfy the borrowing constraint

(25) $$(1 + r_{t+1})\{(1 - q)K_t - qD_t\} = \phi(1 + A)K_t,$$

where (18) and the second equality of (21) are used to derive (25).

The K_t chosen this way is the best response of the private sector, and the private sector has no incentive to deviate from this. Conjecturing this, the government chooses D_t, and hence $d_t = D_t/K(D_t, r_{t+1})$. Through (23), r_{t+1} is a function of d_t, and hence there is a one-for-one mapping from D_t to d_t. The government can control d_t by controlling D_t.

---

[8] Note that $s_t = \tau_t \frac{C_t}{K_{t-1}} - z$, where $z = Z_t/K_{t-1}$, which is assumed to be constant over time.

A competitive equilibrium of this economy is defined by the set of sequences $\{g_t, r_t,\ s_t\}_{t=1}^{\infty}$ that satisfies three equations (20), (23), (24), given the set of initial conditions $\{K_0, W_0^E\}$ and the policy rule $\{d_t\}_{t=0}^{\infty}$.

The dynamic equilibrium is solved in the following. Given the sequence $\{d_t\}_{t=0}^{\infty}$, (23) leads to the sequence $\{r_t\}_{t=0}^{\infty}$. Given the sequences $\{d_t, r_t\}_{t=0}^{\infty}$, (20) characterizes the sequence $\{g_t\}_{t=0}^{\infty}$. Given the sequences $\{d_t, r_t, g_t\}_{t=0}^{\infty}$, (24) leads to the sequence $\{s_t\}_{t=0}^{\infty}$.

When the government aims to set the ratio of public debt to capital to a target, denoted d_{target}, the equilibrium is characterized as the balanced growth path (BGP).

If $d_{target} \geq d_{NF}$, $1 + r_t = 1 + A$, and then $1 + g_t = \beta(1 + A)$ for $t \to \infty$. If $d_{target} < d_{NF}$, there is a unique equilibrium, defined by $\{r^*, g^*\}$, satisfying $1 + r_t = 1 + r^*$, and $1 + g_t = 1 + g^*$, satisfying (20) and (23) for $t \to \infty$. Accordingly, all the variables are constant over time, and the equilibrium features the BGP.

Let us study first the frictionless economy. Interest rates on debt follow $r_t = A$, and equal the return on capital, and ROE. The growth rates satisfy $1 + g_t = \beta(1 + A)$.The standard relation for $r > g$ holds. The allocation is independent of fiscal debt variable d .

Let us turn to the friction economy. Interest rates on debt is less than the return on capital, such that $r_t < A$. The standard relation for $r > g$ may not hold. Interest rates and the growth rate are both dependent on d.

The most important difference between the two economizes concerns fiscal policies. Fiscal policies are neutral to the allocation if $d_t \geq d_{NF}$, but non-neutral if

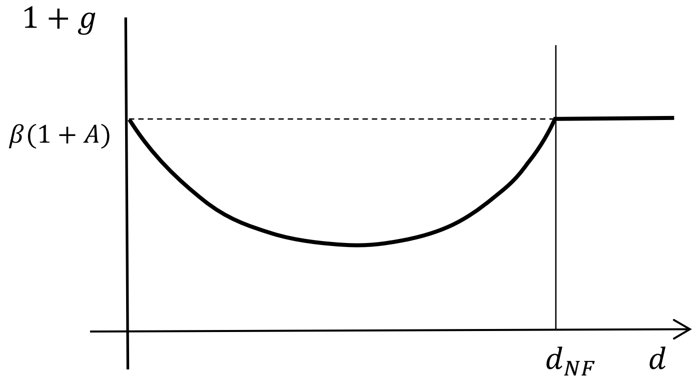
$d_t < d_{NF}$. To show this, we investigate the effect a change in d on economic growth.[9]

Putting (22) into (18), the steady-state growth rate is a function of d:

(26) $$1+g=\beta(1+A)\{\frac{1}{1+d}+\frac{\phi}{1-q-qd}\frac{d}{1+d}\}\equiv G(d).$$

The $G(d)$ function is continuous and satisfies $\lim_{d\to 0} G(d)=\beta(1+A)$, $\lim_{d\to d_{NF}} G(d)=\beta(1+A)$, and $G(d)=\beta(1+A)$ for $d\geq d_{NF}$. The marginal effect of a change in d on growth is $G'(d)=-\frac{\beta(A-r)}{(1+d)^2}+\frac{\beta q(1+r)}{1-q-qd}\frac{d}{1+d}$. The first term is negative if $r<A$, and captures the portfolio effect. This reflects the crowding out of capital. Public debt leads to a rise in interest rates, which is in turn followed by deterring capital growth. The second term captures the liquidity effect that arises through a rise in the average return on wealth.

Figure 3: Public Debt and Economic Growth



[9] Ever since Keynesian economics argued on the Hicks mechanism in the IS-LM analysis, a large literature has argued whether public debt leads to recessions or booms. Models of overlapping generations (see Diamond 1965, Saint-Paul 1992, and others) predict the crowding-out effect of public debt on capital growth. In contrast, when interest rates are so low that public debt plays a role of liquidity, public debt can crowd in capital (see Woodford 1990 and Farhi and Tirole 2012).

Figure 3 illustrates the relation between public debt and economic growth. In the frictionless economy, the growth rate is neutral from public debt. In contrast, in the frictionless economy, the public debt influences significantly the growth rate. The growth rate may be decreasing or increasing in d, depending on the magnitudes of the portfolio effect and liquidity effect. The following proposition summarizes the above argument.

Proposition 1: Suppose that the fiscal policy follows the rule $\{d_t = d_{target}\}_{t=0}^{\infty}$.

(i) If $d_{target} \geq d_{NF}$, the competitive equilibrium realizes an allocation of the frictionless economy, and attains the growth rate $\beta(1 + A)$. Interest rates and the growth rate are independent of d.

(ii) If $d_{target} < d_{NF}$, the competitive equilibrium is subject to financial frictions, and attains the growth rate less than $\beta(1 + A)$. Interest rates and the growth rate are dependent on d.

The non-neutrality of fiscal policies arises from the shortage in the supply of liquidity, private and public debt in our model. The shortage in the liquidity leads to low interest rates (Aiyagari 1994. Aiyagari and McGrattan 1998, and Angeletos *et al*. 2023), and low investment and slow economic growth (Woodford 1990, Holmstrom and Tirole 1998 and Angeletos *et al*. 2023).[10]

The supply of liquidity is effective to eliminate distortions of financial frictions. It will be desirable for private firms to provide liquidity. If firms are able to issue large amount of bonds due to a high ϕ, the supply of liquidity eases financial frictions, and brings the economy to a state of high interest rates and fast economic

[10] In this model the shortage of liquidity leads to low investment. It is contrasted with Aiyagari (1994), in which the absence of insurance markets drives households to save more and lead to over-investment by firms. This result arises from the differences in the sources of friction. Financial frictions arise from capital income risk in ours; they arise from labor income risk in his.

growth. If it is difficult for the private sector to provide additional liquidity, then there is room for the government to provide liquidity.

This argument may tempt us to perceive if the supply of liquidity, either private debt or public debt, is equally desirable for economic growth. This is not true. To show this, consider an experiment of substituting public debt for private debt while keeping the interest rate unchanged. If private debt and public debt are equally desirable for economic growth, the ratio of private debt to public debt B_t/D_t should not have any impact on growth. From (16), a rise in ϕ is associated with a rise in B_t/K_t given the interest rate being constant, and leads to a fall in d_t, and thus a rise in B_t/D_t. At the same time, the growth rate falls. The portfolio shifts from public debt to private debt significantly affects economic growth.

4. Interest rates, growth, and fiscal policies

From the neoclassical point of view, governments are constrained by intertemporal budgets and must run primary surpluses to pay off debt. The economic growth rate is neutral from fiscal policies. But when the economy falls into a state of liquidity shortage, those variables are interrelated through additional channels. We start from the link between interest rates and growth rates.

One interesting feature of this model is that interest rates may fall below growth rates. The following is established.

Proposition 2:

(i) The inequality $r_t < g_t$ holds if only if

(*) $$\phi < \beta(1-q), \quad \text{given } d_t = 0.$$

(ii) The inequality $r_t < g_t$ holds if only if

(**) $\phi\{1 + (1 - \beta)d_t\} < \beta(1 - q - qd_t)$, given $d_t > 0$.

(iii) Suppose that Condition (*) holds. There exists a certain level of debt d_t, below which $r_t < g_t$ holds, and above which $r_t > g_t$ holds.

Proof: (i) and (ii) are obvious from (23) and (20). The proof of (iii) is as follows. The inequality $r_t < g_t$ holds at $d_t = 0$ when Condition (*) holds. Rearranging (20) yields $\frac{1+r_t}{1+g_t} = \frac{1+d_t}{\beta\left(\frac{1+A}{1+r_t}+d_t\right)}$ at $d_t = d_{t-1}$. As r_t is increasing in d_t below A, the RHS is increasing in d_t, and approaches $1/\beta$ (> 1) as the debt increases by enough.
Q.E.D.

Condition (*) means that if borrowing is tightly constrained (low ϕ), entrepreneurs find it difficult to issue private debt, and then the inequality $r_t < g_t$ is more likely to hold. Condition (**) means that if d_t is low, the inequality $r_t < g_t$ is more likely to hold. Taken together, if private debt or public debt, or both are scarce, $r_t < g_t$ is more likely to hold.

One interesting feature is that the return to capital *A* is missing from either condition. When the return to capital rises, both growth rates and interest rates rise proportionally thanks to the linear property of the AK model.

Figure 4: Ratio of Interest Rate to Growth Rate

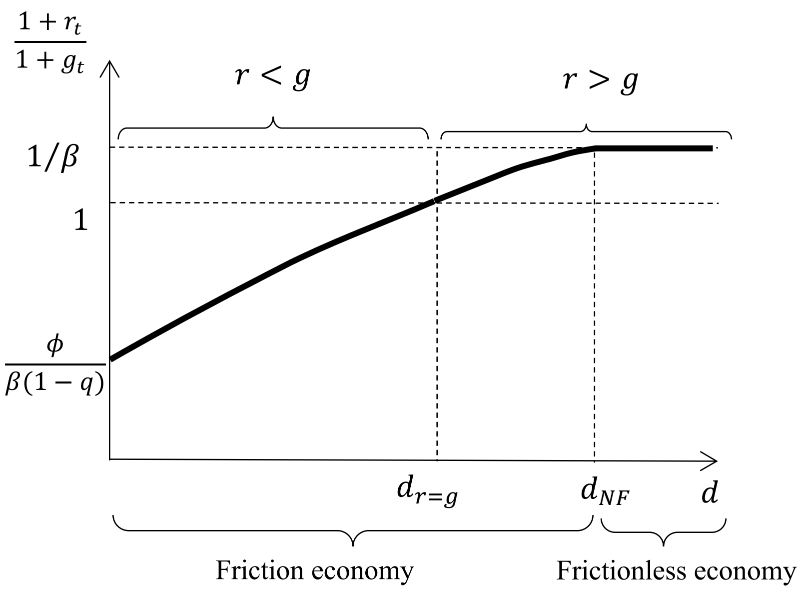


Figure 4 illustrates how the ratio of two rates $\frac{1+r_t}{1+g_t}$ evolves as d_t changes. The debt $d_{r=g} = \frac{\beta(1-q)-\phi}{\beta q+\phi(1-\beta)}$ divides the whole region into the two, $r < g$ and $r > g$. The graph is increasing in d_t until the threshold d_{NF}, above which the borrowing constraint ceases to bind and the ratio of the two rates reaches a constant $1/\beta$.

If we interpret the "natural rate of interest" as the rate at which savings and investments are equal, then interest rates on debt in this model correspond to this rate. Our model explains the coexistence of negative natural rates of interest (see e.g. Krugman 1998) and positive economic growth rates. That does not necessarily mean that the return on capital is negative, but given that entrepreneurs are facing borrowing constraints, it's rather natural to think that the return on capital is positive. Thus, we can explain the reality that interest rates on debt are low and can sometimes be negative, while the growth rate is moderately positive and the return on capital is quite high.

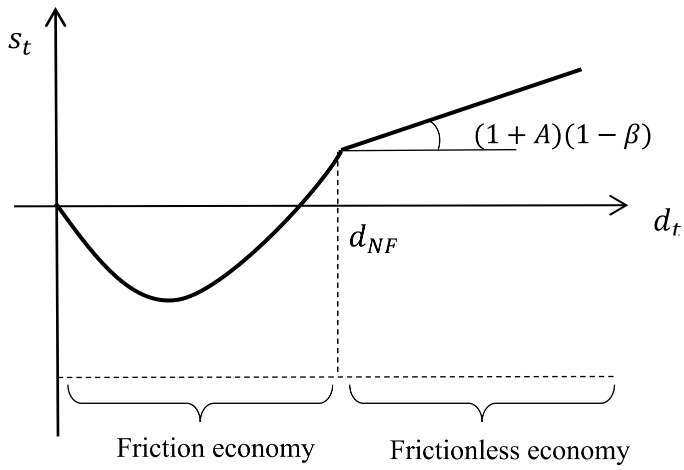
When interest rates on debt are low, a concern on the policy agenda is how to understand the link between public debt and fiscal deficits. Blanchard (2019) proposes a new norm for fiscal policies. He states that if interest rates facing the government is below growth rates, the government is permitted to run deficits.

Here we argue on this issue. Rearranging (24) on the BGP leads to the sustainable level of public debt as $d_t = \frac{s_t}{r_t-g_t}$. If $r_t > g_t$, an implication of the intertemporal approach (e.g., Hamilton and Flavin 1986, and others) is that primary surpluses are necessary for any positive value of public debt. In contrast, if $r_t < g_t$, this equation provides a remarkable implication regarding debt valuation; a positive value of debt is compatible with primary deficits, i.e., $s_t < 0$.

The mechanism is as follows. Suppose to the contrary that the fiscal rule initially dictates a balanced budget. Looking back to (24), the interest payment would increase at the rate of interest, but the ratio of outstanding public debt to capital would decrease at rate of economic growth. If $r_t < g_t$, the former force is dominated by the latter, and primary deficits are permitted to sustain a given level of public debt. If d_t satisfies Condition (**), the government can sustain debt by running primary deficits.[11]

We now consider the link between public debt and fiscal deficits. By incorporating (20) and (23) into (24), we can write fiscal surpluses as a function of public debt. **Figure 5** illustrates how the sustainable primary surplus changes as public debt changes.

Figure 5: Primary Surpluses and Public Debt



The primary surplus s_t has to be positive when the public debt is large, but can be negative when the public debt is small. In the frictionless economy, s_t is linked

[11] We finally discuss the polar case for $r_t = g_t$. When $r_t = g_t$ holds, the balanced budget, i.e., $s_t = 0$, is only consistent with any finite level of public debt. This reasoning is guessed from l'Hopital's rule. The debt dynamics simply follow $d_t = d_{t-1}$, and the government can sustain debt by rollovers.

positively to d_t through $s_t = \frac{d_t}{(1+A)(1-\beta)}$. The government has to run the surpluses to sustain debt. In contrast, in the friction economy, as public debt increases, the primary surpluses are first negative, and later reverts to positive. The government can run sustainable deficits, reflecting that interest rates on debt falls below growth rates when the debt is small. When the debt is large, the government has to run primary surpluses to pay off the debt. Note that the tax rate is also dependent on the public debt.[12]

Public debt affects interest rates on debt and economic growth. Fiscal policies are not neutral. By controlling public debt, the government chooses primary surpluses and at the same time the economic growth rate. A change in public debt is followed by changes in interest rates and growth rates, and determines the sustainable fiscal balance. Remarkably, an economy of fast growth is linked to high interest rates and primary surpluses, whereas an economy of slow growth is linked to low interest rates and primary deficits.

Facing low interest rates, the government would be tempted to believe that the fiscal stance is not a constraint on faster economic growth from the Keynesian point of view. This may be true for a short period of time, but not for long. If the government is far-sighted, it will soon realize that it is impossible to enjoy all of low interest rates, deficits, and fast growth.

[12] The tax rate satisfies $\tau = \frac{\{1+r-(1+g)\}d+z}{(1-\beta)(1+A)+\{1+g-\beta(1+r)\}d-z}$. The tax rate tends to be low when $r < g$, and high as when $r < g$. The tax rate is the highest when d_t arrives at d_{NF}.

5. Which Regime? Applying to the Japanese economy

The Japanese government has run primary deficits ever since 1990s, but the public debt has been sustained. Our model explains that the government can sustain debt by running primary deficits if the economy stays at the region of the friction economy. Our primary concern is in which region the Japanese economy stays between a friction or a frictionless economy.

Our first criterion is to compare between r and g. Graphs in **Figure 1C** depict the real GDP growth rate and the real interest rate on government bonds at 10-years maturity. We find two intervals of period, when interest rates fall below the growth rates. The first interval is the late 1980s, and the second is the period since 2013 when QE started. In the former, low interest rates seem associated with the asset bubble boom (See for example, Farhi and Tirole 2012). We may safely judge that the economy shifted from a frictionless to a friction economy around 2013.

Another criterion is to calculate the threshold d_{NF} and compare it to the actual debt d. To construct d_{NF}, we use the data and our model. The construction of d_{NF} and data source are shown in **Appendix B**.

Figure 6: d and d_{NF}

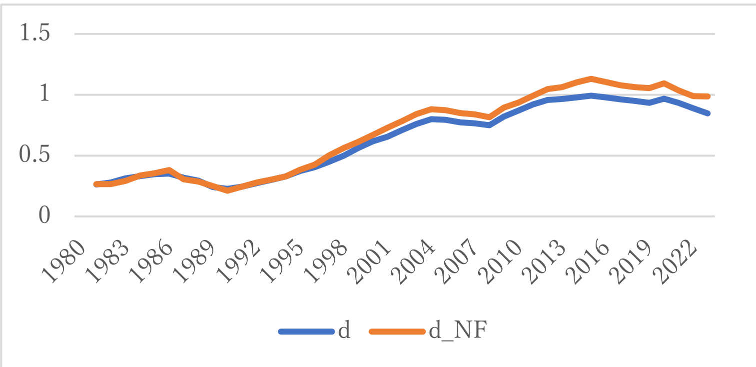


Figure 6 illustrates the time series of d_{NF} for the period for 1981-2023. The calculated series show an increasing trend from the mid-1990s. This tendency suggests that q or ϕ, or both are decreasing over time.

Figure 7: Constructed q

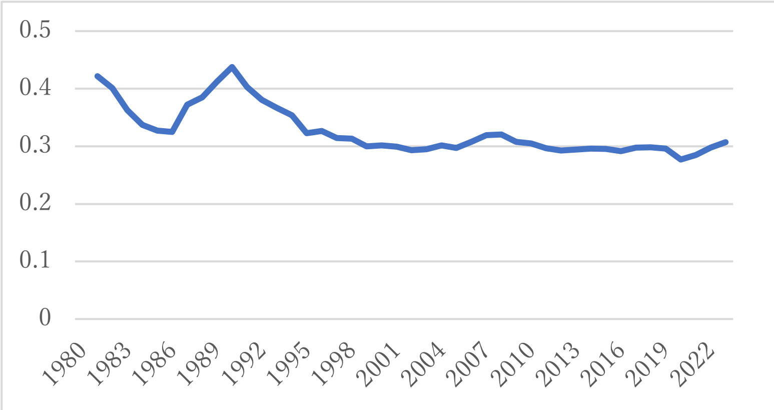
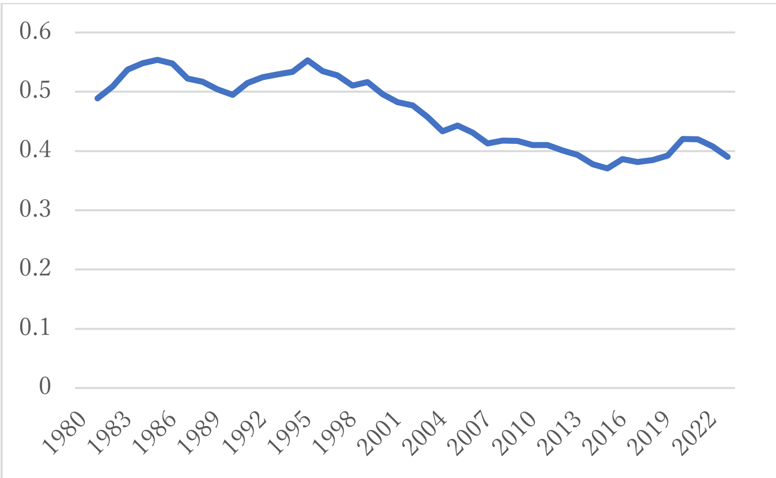


Figure 8: Constructed ϕ



As **figure 7** shows, the calculated q is pretty high around 0.4 in the 1980s, but low around 0.3 after the mid of 1990s. This tendency for the decline in the growth opportunity is roughly consistent with the decline in the GDP growth rates. As **figure 8** shows, the calculated ϕ is pretty high around 0.5 in the 1980s, and starts to fall since the mid of 1990s until recently, which will reflect the secular contraction of the bank lending.[13] A simple calculation illustrates the fall in d_{NF} over time. In the 1980s, both values were high, such that $q = 0.4$ and $\phi = 0.5$, and thus $d_{NF} = (1 - q - \phi)/q = 0.25$. In 2020, both values fell, such that $q = 0.3$ and $\phi = 0.4$, and d_{NF} rose up to unity.

As **figure 6** shows, the two graphs of d and d_{NF} almost coincide with each other until around 1995, which shows that the economy was in the region of a frictionless economy until then. In contrast, in the period after 1995, the graphs diverge; the actual debt d_t is short of the threshold, and its difference is wider over time. The economy falls into a region of a friction economy.

Summing up these arguments, the Japanese economy has shifted from an a frictionless to a friction economy in somewhere between 1995 and 2013.

6. Evaluating Fiscal Policies

In this section we study how fiscal policies are operative when the economy stays at the region of a friction economy. In the past two decades, the government has run primary deficits (see **Figure 1A**) and the ratio of public debt to GDP has increased (see **Figure 1B**).[14]

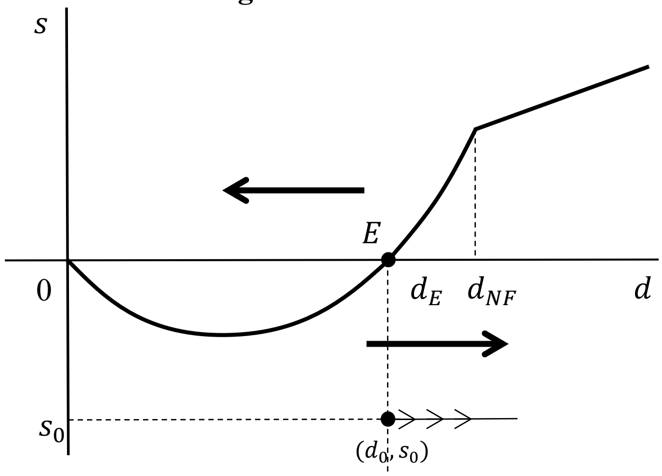
This implies that the equilibrium is apart from the BGP, and the debt is explosive unless primary surpluses improve. In **Figure 8A**, the debt is stable if the equilibrium stays on the curve, but the debt is explosive if the equilibrium is below the curve, and implosive if the equilibrium is above the curve. It will be natural to think that the Japanese economy has initially stayed at (d_0, s_0), that lies below the

[13] The reasons for the contraction of the bank lending are the depreciation of land prices used for collateral, dealing with non-performing loans, and the tighter bank regulation.
[14] In the model primary surpluses and public debt are divided by capital, but when the model follows the standard AK model, it is straightforward to see that the model's implications are directly applied to the real world when primary surpluses and public debt are divided by GDP.

curve; the primary surplus is negative, namely $s_0 < 0$. Without loss of generality, the initial debt d_0 is assumed to be set at the same as d_E.[15]

Restoring fiscal soundness has been an important policy agenda. Since 2005 around, the policy target was to set the primary surplus above zero, and to make the ratio of debt to GDP no more than the original level.

Figure 8A



We evaluate this policy using our model. Assume that the government announces a policy that targets $s = 0$. If this policy is successful, the equilibrium arrives at E on the curve, and the government sustains debt under the balanced budget.

In reality, this policy required a substantial increase in the consumption tax rate, but citizens disagreed to it. The primary surpluses improved, but did not reach the target for $s = 0$. The primary surpluses remained negative.

When this policy fails, the debt is not sustained unless another element is combined. In fact, a fall in real interest rates on debt helped the debt to stabilize.[16]

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[15] When d_0 equals d_E, the dynamics of public debt are simple. If d_0 is different from d_E, the analysis is a little complicated, but qualitatively the same.

[16] The deflation ended and inflation has come since 2013 when the BOJ took a policy of QQE. The BOJ targeted zero nominal interest rate on government bonds at 10 years maturity, and purchased those assets from the markets. The real interest rate fell. In addition, the external shock accelerated inflation since 2022, and the real interest rate fell furthermore.

We represent the fall in real interest rates by a policy of repressing the private debt market and induce investors to shift their portfolio toward public debt away from private debt. Let $1 + r_t^B$ denote the return on private debt, and let χ_t denote the wedge to the return. The return on private debt inclusive of the wedge is represented by $(1 + r_t^B)/(1 + \chi_t)$, and tends to be low when the wedge χ_t is positive. The no arbitrage condition between private and public debt implies $(1 + r_t^B)/(1 + \chi_t) = 1 + r_t$, where r_t denotes the return on public debt. Using (22), the interest rate on debt satisfies

(27) $$(1 + \chi_{t+1})(1 + r_{t+1}) = \frac{\phi(1+A)}{1-q-qd_t}.$$

A wedge operates to induce the interest rate r_t to fall.

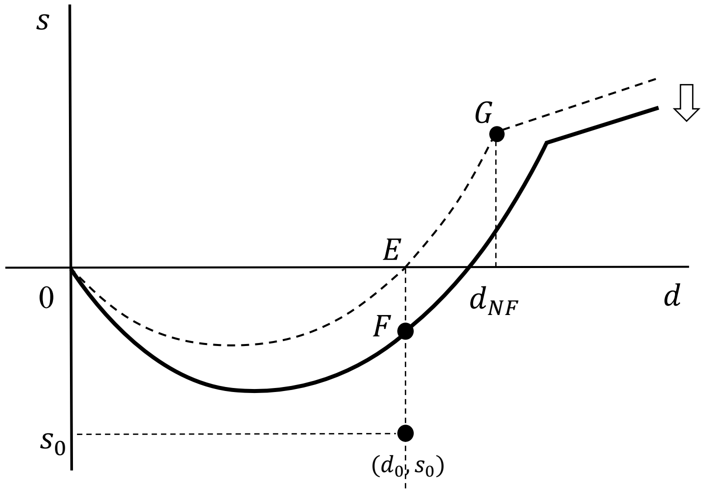
Assume that the government starts the policy of repressing the private bond market, which is captured by a permanent increase in χ_t. We conduct the analysis by assuming that the quantity of public debt remains unchanged before and after this policy. Notably, the public debt outstanding changed by Quantitative Easing Policy (QE), but what QE did was to change the composition of government liabilities, from public bonds at longer maturities to the reserves in the BOJ account.

As **Figure 8B** illustrates, this policy effect is represented as a downward shift of the curve. See **Appendix C** for the derivation. Now the equilibrium arrives at *F*; primary surpluses are negative, but the debt is sustainable.

Our primary question is how to evaluate this consequence that a low-interest-rate policy helped the debt sustainability. We evaluate it from the dimension of growth rate. Comparing growth rates between two equilibria, *E* and *F*, the equilibrium *F* realizes a slower growth rate than *E*. Instead of allowing for primary deficits, the economy suffers from slow economic growth.

What policy realizes the highest growth? The best policy is to attain positive primary surpluses and the debt level at d_{NF}. At the point *G*, the economy realizes the highest growth rate.

Figure 8B



Comparing among three different policies, the equilibrium *G* realizes the largest primary surpluses and the highest growth, whereas the equilibrium *F* realizes the smallest primary surpluses and the lowest growth. Table 1 summarizes this argument.

Table 1: Comparison of three different policies

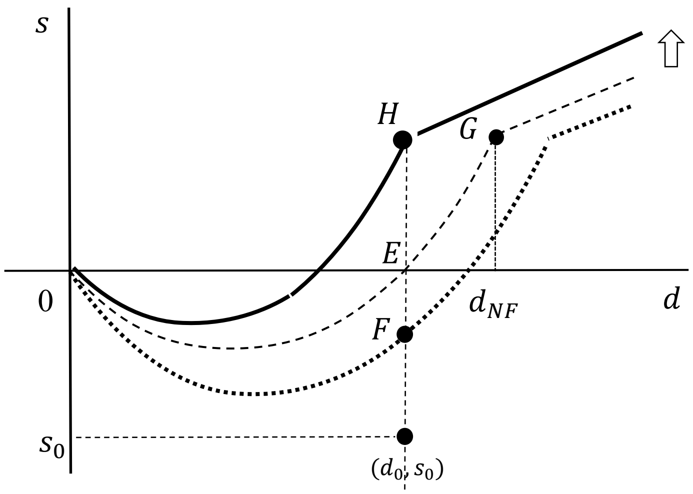
	Targeted (point *E)*	Realized (point *F*)	Desirable (Pont *G*)
Primary surplus (s)	0	negative	positive
Public debt (d)	d_E	d_E	$d_{NF}(> d_E)$
Growth rate (g)	medium	low	high
Interest rate (r)	medium	low	high

The choice faced by the government is either the set of deficits and slow growth or the set of surpluses and fast growth. The choice of deficits and fast growth is not sustainable. A short-sighted government might be tempted to enjoy low interest rates, deficits, and fast growth, but a rational government will realize that it is impossible to have them all in the long run. The Japanese government may advocate "growth first, fiscal consolidation next.", but has to find it impossible to realize fast growth by keeping deficits and low interest rates long.

Finally, we comment on one important thing beyond fiscal policies. One concern about this table is that at the “desirable” equilibrium *G* the public debt is higher than other equilibria. In this model, slow growth arises from the liquidity shortage, and thus the larger public debt leads to faster growth. On the other hand, we would consider if the smaller public debt attains the highest growth rate.

We could remember the fact that private debt can solve the shortage in liquidity as well as public debt. Private debt increases if borrowing constraints are relaxed. Consider an experiment that increases pledgeability. In **Figure 8C**, the new curve is illustrated to shift upwardly, and the equilibrium *H* is realized. It realizes the highest growth rate, given the initial level of debt d_0. Instead of relying on further public debt, the economy realizes the fastest growth by relying on the private debt. This experiment implies that if financial markets function well, the small public debt is enough to realize the good economy.

Figure 8C



7. Conclusion

This paper has established a growth theory that enable us to study fiscal policies not only in the neoclassical regime but also in a regime of liquidity shortage. When falling in the regime of liquidity shortage, interest rates on debt can fall below the economic growth rate, and then the government can sustain debt by running primary deficits. Low interest rates on debt arise from the shortage in liquidity, and thus those low rates are associated with low investment and slow economic growth. The choice faced by the government is either the set of deficits and slow growth or the set of surpluses and fast growth.

We show that the current Japanese economy falls into a region of liquidity shortage. We evaluate fiscal policies at aiming fiscal surpluses above zero from the perspective of our model.

One promising direction for future research is to study debt sustainability. Unlike the existing literature, this model provides a framework that covers not only the neoclassical norm of $r > g$ but also the new norm of $r < g$, proposed by Blanchard (2019). If the government is allowed to run deficits due to low interest rates, how the analysis of debt sustainability will change is of great interest.

Another interesting direction is to study the welfare of different fiscal policies in our model. The welfare analysis leads us to approach the "optimal debt problem". Studying the optimal public debt under low interest rates is of great interest.

Appendix

A: *Saving rate of entrepreneurs*

The problem of entrepreneurs is defined by

$V_j(w_{t-1}) = \max_{w_t} \log c_t^j + \beta q V_E(w_t) + \beta(1-q)V_I(w_t)$, $(j = EA, IE)$ subject to $(1 + r_t^j)w_{t-1} = c_t^j + w_t$.

The first-order condition is $\frac{1}{c_t^j} = \beta q V_E{}'(w_t) + \beta(1-q)V_I{}'(w_t)$. By the envelope, $V_j{}'(w_{t-1}) = \frac{1+r_t^j}{c_t^j}$. Assume that consumption and savings are linear in wealth; $c_t^j = (1-s_t)\left(1+r_t^j\right)w_{t-1}$, and $w_t = s_t\left(1+r_t^j\right)w_{t-1}$. The first-order condition is rewritten as $\frac{1}{(1-s_t)\left(1+r_t^j\right)w_{t-1}} = \beta\frac{1}{(1-s_{t+1})\,w_t}$. Using $w_t = s_t\left(1+r_t^j\right)w_{t-1}$, $\frac{s_t}{(1-s_t)} = \beta\frac{1}{(1-s_{t+1})}$. $s_t = \beta$ at the steady state when $s_t = s_{t+1}$. Q.E.D.

B: ***Construction of*** $\mathbf{d_{NF}}$ ***and the data***

For the calculation of d_{NF}, we use equation ($), $d_{NF} = \frac{1-q-\phi}{q}$. In constructing q, we use (12), implying that $q = \frac{W^{EA}}{W^E}$, namely, the share of equity in the total wealth equals q. From the relation $W^{EA} = K - B$ (equation (13)) and the relation $W^E = K + D$ (equation (5)), we finally derive that $q = \frac{K-B}{K+D}$. Next in constructing ϕ , we use equation (15), $\phi = \frac{(1+r)B}{(1+return\ on\ capital)K}$.

We construct data series K, B, D, and the return on capital, using SNA datasets (data source: Cabinet Office, Japan).

K: Non-financial assets and foreign direct investment held by non-financial corporations as assets. Non-financial assets consist of produced assets and non-produced assets such as land.

B: Loans and debt securities (industrial securities, external securities issued by residents, and commercial paper) held by non-financial corporations as liabilities.

D: we consider the government debt as the overall debt of the government and calculate as the total of stock of financial liabilities of general government and the total of stock of currency in a whole Japanese economy.

To construct the return on capital, we consider in the following way. It is appropriate to think that output of the AK model is divided into the capital income and the labor income. Assuming that the capital income share in the total output is constant, denoted α, αAK goes to the capital income, and $(1-\alpha)AK$ goes to the labor income.

We calculate the return on capital as $\alpha A-\delta$ where δ is the deprecation rate of capital. We set $\alpha=0.35$ and $\delta=0.08$, which are commonly used in the prior research. We calculate A by Y/K, where we use nominal GDP for Y, and use "capital" for K. "Capital" used here is the sum of non-financial assets and foreign direct investment held by non-financial corporations as assets and produced assets held by households (including private unincorporated enterprises) and private non-profit institutions serving households as assets.

For obtain data series of K, B, D and A for 1981-2013, we use two kinds of data set, one is Annual Report on National Accounts for 2023 (2008SNA benchmark year = 2015), which covers the period 1994-2013 and the other is Annual Report on National Accounts for 2009 (1993SNA benchmark year = 2000), which covers the period 1980-2003. In creating the long-term series from 1980-2023, we use figures from the latter dataset from 1980 to 1993 and figures from the former data set from 1994 to 2023. The pre-1993 data are adjusted using a linking factor calculated as the ratio of overlapped values between the former and the latter series.

In terms of r, we use the interest rate on 10-years Japanese bond from Ministry of Finance Japan.

C: *Effects of financial repression*

By incorporating (20) and (23) into (24), we can write fiscal surpluses as a function of steady-state public debt;

$$s = (1 + r)d - (1 + g)d$$

$$= \frac{\phi(1+A)}{1-q-qd}d - \beta(1 + A)\{\frac{1}{1+d} + \frac{\phi}{1-q-qd}\frac{d}{1+d}\}d.$$

When (20) is replaced by (27), it is rewritten as

$$s = \frac{\phi(1 + A)}{(1 + \chi)(1 - q - qd)}d - \beta(1 + A)\{\frac{1}{1 + d} + \frac{\phi}{(1 + \chi)(1 - q - qd)}\frac{d}{1 + d}\}d$$

Since $\beta\frac{d}{1+d} < 1$, the LHS is decreasing in χ.

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