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# Asset pricing interpretations of the primary fiscal balance: The case of Japan

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# Asset pricing interpretations of the primary fiscal balance: The case of Japan[1]

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**Abstract**: Chien et al. (2025) state controversially that the Japanese government has exploited available arbitrage opportunities, and that it has backed its huge liabilities by high $\beta$ financial assets and fiscal claims. A major empirical ground for their statement is that the government reduced its net liabilities substantially by earning enormous capital gains from domestic and foreign risky assets from the mid-2010s through to the mid-2020s. Employing a net liability version of the primary fiscal balance (PFB), we prove theoretically that only if arbitrage opportunities are available to the government, high $\beta$ assets and claims the government is currently holding may improve the discounted PFB. However, we empirically demonstrate that there is no strong evidence for the presence of arbitrage opportunities for the government, and that its net positions bear at most moderate $\beta$. These theoretical and empirical findings contradict Chien et al.'s statement.

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## 1. Introduction

Chien et al. (2025) state controversially that the Japanese government has exploited available arbitrage opportunities and has backed its gross liabilities by high $\beta$ gross financial assets and fiscal claims, whose random payoffs evolve according to a net liability version of the primary fiscal balance (PFB). That is, its huge gross liabilities are still sustainable with such a high $\beta$ claim on the fiscal surplus as well as a large amount of risky assets with high $\beta$, which are held by various bodies of the government. Their statement could justify gigantic public debt without resorting to the growth rate exceeding the interest rate ($g > r$) argument as in Blanchard (2019) and others, or the violation of the transversality condition as in Saito (2021) and others. This paper examines their statement with extreme care, first theoretically using a standard asset pricing model, and second empirically using a standard $\beta$-CAPM regression.[3]

A major empirical ground for their statement is that the Japanese general government (GG), consisting of the central and local governments, and the social security funds, successfully reduced its net liabilities by earning enormous amounts of capital gains from risky assets, both domestic and foreign, from the mid-2010s through to the mid-2020s. According to the Flow of Funds Accounts compiled by the Bank of Japan (BoJ),[4] as shown in Figure 1-1, the gross liabilities (red column in negative numbers) expanded from the late 1990s to the early 2020s, but the net liabilities (black line) ceased to increase in the early 2010s and started to decline from the late 2010s.

(insert Figure 1-1)

As Figure 1-2 demonstrates, this trend is more obvious when measured relative to nominal gross domestic product. A major reason for the behavioral difference between gross and net liabilities is that most capital gains from risky assets are unrealized and are not included in the revenue side of the conventional PFB, without contributing to a decline in gross liabilities. However, such unrealized capital gains are reflected in the market valuation of gross assets, which are subtracted from gross liabilities in deriving net liabilities.

(insert Figure 1-2)

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[3] Huang and Litzenberger (1988) provide one of the most intensive and extensive discussions of $\beta$-CAPM.

[4] See Bank of Japan (1998–2024).

The government sector indeed allocated a significant portion of gross assets to domestic and foreign risky investment. Accordingly, the gross asset side changed quite differently from the gross liability side, depending on how equity returns and exchange rates behaved in the form of unrealized returns. As shown in Figure 2-1, the central government, particularly the foreign exchange fund special account, started to invest in foreign assets and domestic equities from the early 2000s, while, as shown in Figure 2-2, the social security funds started to shift from Japanese government bonds (JGBs) to domestic equities and foreign assets in the late 2000s. Regarding the gross assets of the BoJ (as another body of the integrated government (IG)) in Figure 2-3, its investment shifted from treasury bills (short-term JGBs) to long-term JGBs, and domestic equities from the early 2010s. But the BoJ's investment in foreign assets remained negligible.

(insert Figure 2-1)

(insert Figure 2-2)

(insert Figure 2-3)

In this paper, we redefine the PFB according to not gross, but net liability dynamics. As in Jiang et al. (2024) and Chien et al. (2025), we treat such a net liability version of the PFB as a random payoff from the government's investment in risky assets, and its claim on the fiscal surplus, and discount it by a stochastic factor to derive the present value of the future PFB. Stepping back from a general setup in Jiang et al. (2024), we adopt, as a concrete simple specification, a stochastic discount factor implicit in $\beta$-CAPM. Another important difference between Chien et al. and ours is that the former targets the expected PFB, but the latter focuses on the discounted PFB.

We challenge the abovementioned statement in Chien et al. both theoretically and empirically. In our theoretical part, we demonstrate that given no arbitrage opportunity available to the government, high $\beta$ assets and claims, both of which are carried by the government, improve the expected PFB, which is their focus, but they have no impact on the discounted PFB, which is our focus. More concretely, if any asset is priced properly under some stochastic discount factor, then regardless of the quantity of high $\beta$ assets held by the government, these holdings do not affect the discounted PFB. Conversely, we prove that if the government can exploit some arbitrage opportunities, then high $\beta$ assets may improve not only the expected PFB, but also the discounted PFB. More specifically, if the Euler equation does not hold with respect to its excess returns, some or all parameters of the $\beta$-CAPM regression for risky excess returns may have effects on the discounted PFB.

In our empirical part, we carefully explore whether arbitrage opportunities are available to the government, and how high is the value of $\beta$ on the net financial and fiscal positions of the government. First, there is no strong evidence for the presence of arbitrage opportunities available to the government. Second, even if all parameters of the $\beta$-CAPM regression are considered to the fullest, together with the assumed presence of arbitrage opportunities for the government, the estimated expected PFB bears at most a moderate $\beta$, and still yields low or negative expected returns on average. As Chien et al. point out as their major empirical ground, the expected return was indeed positive from the mid-2010s through to the mid-2020s, but that positive return emerged not because of high $\beta$ assets and claims, but just because of high market returns. With the BoJ's net assets included, these results do not change.

Given the above theoretical results and empirical findings, the enormous liabilities of the Japanese government are not backed by a high $\beta$ of either financial assets or fiscal claims on future fiscal surpluses, and they remain unfunded despite active risky investment by various bodies of the IG. In conclusion, it is difficult to immediately justify the abovementioned statement by Chien et al.

This paper is organized as follows. In Section 2, we define the PFB according to the net liability dynamics and explore theoretically how parameters of the $\beta$-CAPM regression affect the present value of the future PFB. In Section 3, using the Flow of Funds Accounts, we compute the time series of a net liability version of the PFB, and estimate the present value of the future PFB. In Section 4, given our theoretical and empirical exercises, we conclude the paper.

## 2. Net liability version of the primary fiscal balance

### 2.1. Constant discounting as risk neutrality

The PFB of the GG is usually determined according to the following gross liability dynamics equation:

$$\Delta D_t^{GG} = -[(T_t^{GG} + r_t^{GG} A_{t-1}^{GG}) - (G_t^{GG} + \Delta A_t^{GG})] + r_t^D D_{t-1}^{GG}, \tag{1}$$

where $D_t^{GG}$ and $A_t^{GG}$ are gross liabilities and gross assets, both of which are outstanding in nominal terms at the end of time $t$. On the nominal expenditure side, $G_t^{GG}$ is defined as government expenditure, which excludes interest payments ($r_t^D D_{t-1}^{GG}$) and net asset purchases ($\Delta A_t^{GG}$). However, the nominal revenue side consists of tax revenue ($T_t^{GG}$) and returns from gross assets ($r_t^{GG} A_{t-1}^{GG}$). $r_t^D$ and $r_t^{GG}$ denote nominal interest rates on short-term JGBs and nominal yields on gross assets. Thus, the PFB is defined as:

$$PFB_t^{GL} = (T_t^{GG} + r_t^{GG} A_{t-1}^{GG}) - (G_t^{GG} + \Delta A_t^{GG}). \tag{2}$$

There are two potential problems in using the above gross liability version of the PFB. First, returns from gross assets record only realized returns ($r_t^{GG,R} A_{t-1}^{GG}$), and do not include any unrealized capital gains or losses ($r_t^{GG,UR} A_{t-1}^{GG}$). Second, net asset purchases ($\Delta A_t^{GG,R}$) are recorded only on a purchase and sale basis, and do not reflect market valuations.

This paper instead proposes a *net* liability version of the PFB with due consideration to unrealized returns from gross assets ($r_t^{GG,UR} A_{t-1}^{GG}$), as well as market valuations of net liabilities ($D_t^{GG} - A_t^{GG}$) and their increments ($\Delta(D_t^{GG} - A_t^{GG})$), in which both $A_t^{GG}$ and $A_{t-1}^{GG}$ are evaluated in terms of market prices. That is, the PFB is determined according to the following net liability dynamics.

$$\Delta(D_t^{GG} - A_t^{GG}) = -\left[(T_t^{GG} - G_t^{GG}) + \left(r_t^{GG,R} + r_t^{GG,UR} - r_t^D\right) A_{t-1}^{GG}\right] + r_t^D (D_{t-1}^{GG} - A_{t-1}^{GG}). \tag{3}$$

Then, a net liability version of the PFB is now defined as:

$$PFB_t^{NL} = (T_t^{GG} - G_t^{GG}) + \left(r_t^{GG,R} + r_t^{GG,UR} - r_t^D\right) A_{t-1}^{GG}. \tag{4}$$

A comparison between (2) and (4) shows that in the latter, net asset purchases ($\Delta A_t^{GG}$) are dropped, but unrealized excess returns ($\left(r_t^{GG,UR} - r_t^D\right) A_{t-1}^{GG}$) are included. One important feature of $PFB_t^{NL}$ is that not gross returns ($r_t^{GG,R} + r_t^{GG,UR}$), which appear in Chien et al.'s net liability dynamics,[5] but excess returns ($r_t^{GG,R} + r_t^{GG,UR} - r_t^D$) show up in the right-hand side of Equation (3) or (4).

Let us assume that agents are risk neutral, and that they discount future nominal payoffs by a constant nominal factor. If the future PFB is discounted by a constant interest rate on short-term JGBs ($r^D$), then Equation (3), which has a recursive structure, is rewritten in a forward-looking manner:

[5] Chien et al. (2025) formulate the net liability dynamics as follows: $D_t - A_t = (G_t - T_t) + (1 + r_t^D) D_{t-1} - (1 + r_t^R + r_t^{UR}) A_{t-1}$.

$$D_{t-1}^{GG}-A_{t-1}^{GG}=E_{t-1}\left\{\frac{\left[(T_t^{GG}-G_t^{GG})+\left(r_t^{GG,R}+r_t^{GG,UR}-r^D\right)A_{t-1}^{GG}\right]+(D_t^{GG}-A_t^{GG})}{1+r^D}\right\}.$$

As a simplifying assumption, constancy of nominal interest rates ($r^D$) abstracts any dynamics from inflation and real interest rates. However, most implications from the current model, particularly related to the discounted PFB, survive without this assumption.

Here, both sides of the above equation are divided by one-period lagged gross liabilities $D_{t-1}^{GG}$. Then, it is further developed in a sequential manner.

$$1-\frac{A_{t-1}^{GG}}{D_{t-1}^{GG}}=\sum_{\tau=0}^{\infty}\mathrm{E}_{t-1}\left\{\frac{1}{(1+r^D)^{\tau}}\frac{D_{t-1+\tau}^{GG}}{D_{t-1}^{GG}}\mathrm{E}_{t-1+\tau}\left[\frac{1}{1+r^D}\frac{(T_{t+\tau}^{GG}-G_{t+\tau}^{GG})+(r_t^{GG,R}+r_t^{GG,UR}-r^D)A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]\right\}+\frac{1}{D_{t-1}^{GG}}\lim_{\tau\to\infty}\mathrm{E}_t\left[\frac{D_{t+\tau}^{GG}-A_{t+\tau}^{GG}}{(1+r^D)^{\tau+1}}\right]. \quad (5)$$

Let us assume for simplicity that (i) $\lim_{\tau\to\infty}\mathrm{E}_t\left[\frac{D_{t+\tau}^{GG}-A_{t+\tau}^{GG}}{(1+r^D)^{\tau+1}}\right]=0$ by a transversality condition. By assumption (i), the current valuation of the net liabilities is equal to the present value of the future PFB. As a corollary of assumption (i), we assume that (ii) the real balance of $D_{t-1}^{GG}$ remains constant around a steady state; that is, $D_{t-1}^{GG}$ grows at the average inflation rate $\pi$, or $D_{t-1+\tau}^{GG}=(1+\pi)^{\tau}D_{t-1}^{GG}$. In addition, (iii) $\pi$ is lower than $r^D$, and a real interest rate $\rho^D$ is positive ($\rho^D=r^D-\pi>0$).

Then, Equation (5) is further rewritten as:

$$1-\frac{A_{t-1}^{GG}}{D_{t-1}^{GG}}=\sum_{\tau=0}^{\infty}\mathrm{E}_{t-1}\left\{\left(\frac{1+\pi}{1+r^D}\right)^{\tau}\mathrm{E}_{t-1+\tau}\left[\left[\frac{1}{1+r^D}\left(\frac{T_{t+\tau}^{GG}-G_{t+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right)+\left(r_{t+\tau}^{GG,R}+r_{t+\tau}^{GG,UR}-r^D\right)\frac{A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]\right]\right\}$$

$$=\frac{1}{1+r^D}\sum_{\tau=0}^{\infty}\left\{\left(\frac{1+\pi}{1+r^D}\right)^{\tau}\left[\mathrm{E}_{t-1}\left(\frac{T_{t+\tau}^{GG}-G_{t+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right)+\mathrm{E}_{t-1}\left[\frac{A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\mathrm{E}_{t-1+\tau}\left(r_{t+\tau}^{GG,R}+r_{t+\tau}^{GG,UR}-r^D\right)\right]\right]\right\}. \quad (6)$$

Taking the unconditional expectation for both sides, the present value of $PFB_t^{NL}$ relative to $D_{t-1}^{GG}$ leads to:

$$1-\mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right)=\frac{1}{\rho^D}\left\{\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)+\mathrm{E}\left[\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\left(r^{GG,R}+r^{GG,UR}-r^D\right)\right]\right\},$$

where $\rho^D=r^D-\pi$. In this way, the unconditional expectation of $PFB_t^{NL}$ relative to $D_{t-1}^{GG}$ corresponds to:

$$\mathrm{E}\left(\frac{PFB^{NL}}{D_{-1}^{GG}}\right) = \mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right) + \mathrm{E}\left[\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\left(r^{GG,R} + r^{GG,UR} - r^{D}\right)\right]. \tag{7}$$

However, taking risk neutrality seriously as the underlying assumption, average excess returns or risk premia should be equal to zero. That is, $\mathrm{E}_{t-1+\tau}\left[\left(r_{t+\tau}^{GG,R} + r_{t+\tau}^{GG,UR} - r^{D}\right)\right]$ on the right-hand side of Equation (6) or (7) degenerates to zero. Accordingly, the unconditional expectation of $PFB_t^{NL}$ relative to $D_{t-1}^{GG}$ turns out to be $\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)$, and its present value relative to $D_{t-1}^{GG}$ reduces to:

$$1 - \mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right) = \frac{1}{\rho^{D}}\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right). \tag{8}$$

Under constant discounting as risk neutrality, the discounted relative $PFB_t^{NL}$ as well as its present value $(1 - \mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right))$ have nothing to do with how high excess returns are from GG's risky investment. The only source of positive cash flows for repaying gross liabilities is the average surplus in the conventional PFB, or $\mathrm{E}\left(T^{GG} - G^{GG}\right) > 0$.

### 2.2. Stochastic discounting as risk aversion

We now consider risk aversion instead of risk neutrality to evaluate the possible impacts of excess returns from GG's risky investment on the present value of $PFB_t^{NL}$. We adopt a time-varying nominal factor, which discounts future positive (negative) nominal payoffs more (less) heavily. Concretely, the following stochastic discount factor implicit in $\beta$-CAPM is chosen for this purpose:

$$DF_t = \frac{1 - \frac{\mathrm{E}\left(r^{m}-r^{D}\right)}{\mathrm{E}\left(r^{m}-r^{D}\right)^{2}}\left(r_t^{m}-r^{D}\right)}{1+r^{D}}, \tag{9}$$

where $r_t^m - r^D$ is the excess market return, and it is assumed to follow:

$$r_t^m - r^D = \mathrm{E}\left(r^m - r^D\right) + \epsilon_t^m, \tag{10}$$

where $\epsilon_t^m$ is white noise. Under this simplifying assumption, only $\epsilon_t^m$ represents the aggregate market risk.

The above stochastic discount factor $DF_t$ is linear in a random payoff $r_t^m - r^D$. In addition, it is negatively corelated with any excess return $r_t^i - r^D$; that is, it discounts positive (negative) realizations of excess returns more (less) heavily.

By construction, $DF_t$ is orthogonal to excess market returns $r_t^m - r^D$ on average:

$$\mathrm{E}_{t-1}[DF_t(r_t^m - r^D)] = \mathrm{E}_{t-1}\left[\frac{1-\frac{\mathrm{E}\left(r^m-r^D\right)}{\mathrm{E}\left(r^m-r^D\right)^2}(r_t^m-r^D)}{1+r^D}(r_t^m - r^D)\right] = 0. \tag{11}$$

As a legitimate discount factor, $DF_t$ should be orthogonal to any other excess return on asset $i$ $(r_t^i - r^D)$ on average:

$$\mathrm{E}_{t-1}\left[DF_t(r_t^i - r^D)\right] = \mathrm{E}_{t-1}\left[\frac{1-\frac{\mathrm{E}\left(r^m-r^D\right)}{\mathrm{E}\left(r^m-r^D\right)^2}(r_t^m-r^D)}{1+r^D}(r_t^i - r^D)\right] = 0 \qquad \forall\, i. \tag{12}$$

Using $\mathrm{E}_{t-1}(x_t y_t) = \mathrm{Cov}_{t-1}(x_t, y_t) + \mathrm{E}_{t-1}(x_t)\mathrm{E}_{t-1}(y_t)$, we can derive $\beta$-CAPM from Equations (10), (11), and (12) as follows:

$$r_t^i - r^D = \beta^i(r_t^m - r^D) + \eta_t^i, \tag{13}$$

where $\beta^i = \frac{Cov(r_t^i - r^D, r_t^m - r^D)}{Var(r_t^m - r^D)}$, and $\eta_t^i$ is white noise. Note that there is no constant term on the right-hand side of Equation (13).

Let us apply $DF_t = \frac{1-\frac{\mathrm{E}\left(r^m-r^D\right)}{\mathrm{E}\left(r^m-r^D\right)^2}(r_t^m-r^D)}{1+r^D}$ instead of $\frac{1}{1+r^D}$ to discount the future PFB, which is defined by Equation (4). In addition, we maintain assumption (ii), or $D_{t-1+\tau}^{GG} = (1+\pi)^\tau D_{t-1}^{GG}$, and assumption (iii) $\rho^D = r^D - \pi > 0$:

$$1 - \frac{A_{t-1}^{GG}}{D_{t-1}^{GG}} = \sum_{\tau=0}^{\infty} \mathrm{E}_{t-1}\left\{(1+\pi)^\tau \prod_{i=0}^{\tau}(DF_{t+i})\left[\frac{(T_{t+\tau}^{GG} - G_{t+\tau}^{GG}) + \left(r_{t+\tau}^{GG,R} + r_{t+\tau}^{GG,UR} - r_{t+\tau}^D\right)A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]\right\}$$

$$= \sum_{\tau=0}^{\infty} \mathrm{E}_{t-1}\left\{(1+\pi)^{\tau}\prod_{i=0}^{\tau-1}(DF_{t+i})\,\mathrm{E}_{t-1+\tau}\left[DF_{t+\tau}\frac{(T_{t+\tau}^{GG}-G_{t+\tau}^{GG})+\left(r_{t+\tau}^{GG,R}+r_{t+\tau}^{GG,UR}-r_{t+\tau}^{D}\right)A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]\right\}.$$

Using Equation (13), $r_{t+\tau}^{GG,R}+r_{t+\tau}^{GG,UR}-r_{t+\tau}^{D}$ is replaced with $\beta^{GG}(r_{t+\tau}^{m}-r^{D})+\eta_{t+\tau}^{GG}$.

$$1-\frac{A_{t-1}^{GG}}{D_{t-1}^{GG}} = \sum_{\tau=0}^{\infty} \mathrm{E}_{t-1}\left\{(1+\pi)^{\tau}\prod_{i=0}^{\tau-1}(DF_{t+i})\,\mathrm{E}_{t-1+\tau}\left[DF_{t+\tau}\frac{(T_{t+\tau}^{GG}-G_{t+\tau}^{GG})+(\beta^{GG}(r_{t+\tau}^{m}-r^{D})+\eta_{t+\tau}^{GG})A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]\right\}. \quad (14)$$

Noting that $\mathrm{E}_{t+\tau-1}[DF_{t+\tau}(\beta^{GG}(r_{t+\tau}^{m}-r^{D})+\eta_{t+\tau}^{GG})]=0$ by Equation (11), we simplify Equation (14) as:

$$1-\frac{A_{t-1}^{GG}}{D_{t-1}^{GG}} = \sum_{\tau=0}^{\infty} \mathrm{E}_{t-1}\left\{(1+\pi)^{\tau}\prod_{i=0}^{\tau-1}(DF_{t+i})\mathrm{E}_{t-1+\tau}\left[DF_{t+\tau}\frac{T_{t+\tau}^{GG}-G_{t+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]\right\}. \quad (15)$$

A fundamental reason for the absence of excess returns in Equation (15) is that the past realizations of capital gains and losses have already been reflected in the evaluation of current gross assets $A_{t-1}^{GG}$. In terms of the future possibilities of capital gains and losses, however, positive (negative) excess returns are discounted more (less) heavily under stochastic discounting, and the discounted excess returns degenerate to zero. Accordingly, $\beta^{GG}(r_{t+\tau}^{m}-r^{D})$ has no impact on the discounted PFB.

Regressing $\frac{T_{t+\tau}^{GG}-G_{t+\tau}^{GG}}{D_{t-1+\tau}^{GG}}$ on $r_t^{m}-r^{D}$, defined by Equation (10), leads to:

$$\frac{T_{t+\tau}^{GG}-G_{t+\tau}^{GG}}{D_{t-1+\tau}^{GG}} = \alpha^{T-G}+\beta^{T-G}(r_{t+\tau}^{m}-r^{D})+\eta_t^{T-G}, \quad (16)$$

where $\beta^{T-G}=\frac{Cov\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}},r^{m}-r^{D}\right)}{\mathrm{Var}(r^{m}-r^{D})}$, and $\alpha^{T-G}=\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)-\beta^{T-G}E(r^{m}-r^{D})$.

Replacing $\frac{T_{t+\tau}^{GG}-G_{t+\tau}^{GG}}{D_{t-1+\tau}^{GG}}$ by $\alpha^{T-G}+\beta^{T-G}(r_{t+\tau}^{m}-r^{D})+\eta_t^{T-G}$, we obtain:

$$\mathrm{E}_{t-1+\tau}\left[DF_{t+\tau}\frac{T_{t+\tau}^{GG}-G_{t+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right] = \mathrm{E}_{t-1+\tau}\{DF_{t+\tau}[\alpha^{T-G}+\beta^{T-G}(r_{t+\tau}^{m}-r^{D})+\eta_t^{T-G}]\}$$

$$= \mathrm{E}_{t-1+\tau}(DF_{t+\tau})\left[\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)-\beta^{T-G}E(r^{m}-r^{D})\right],$$

thanks to $\mathrm{E}_{t+\tau-1}[DF_{t+\tau}(\beta^{GG}(r^m_{t+\tau}-r^D)+\eta^{GG}_{t+\tau})]=0$ by Equation (11), and $\alpha^{T-G}=\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D^{GG}_{-1}}\right)-\beta^{T-G}E(r^m-r^D)$.

Applying Equation (10), and using $\mathrm{E}(DF)=\left\{1-\frac{[\mathrm{E}(r^m-r^D)]^2}{\mathrm{E}(r^m-r^D)^2}\right\}/(1+r^D)\approx 1/\left\{1+r^D+\frac{[\mathrm{E}(r^m-r^D)]^2}{\mathrm{E}(r^m-r^D)^2}\right\}$ when $\mathrm{E}(r^m-r^D)^2$ is close to $\mathrm{Var}(r^m-r^D)$, Equation (15) is further simplified as:

$$1-\mathrm{E}\left(\frac{A^{GG}_{-1}}{D^{GG}_{-1}}\right)=\left[\left\{1-\frac{[\mathrm{E}(r^m-r^D)]^2}{\mathrm{E}(r^m-r^D)^2}\right\}/(1+r^D)\right]\sum_{\tau=0}^{\infty}\left[\left\{1-\frac{[\mathrm{E}(r^m-r^D)]^2}{\mathrm{E}(r^m-r^D)^2}\right\}(1+\pi)/(1+r^D)\right]^{\tau}\left[\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D^{GG}_{-1}}\right)-\beta^{T-G}E(r^m-r^D)\right]$$
$$\approx\frac{1}{\rho^D+\frac{[\mathrm{E}(r^m-r^D)]^2}{\mathrm{E}(r^m-r^D)^2}}\left[\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D^{GG}_{-1}}\right)-\beta^{T-G}E(r^m-r^D)\right], \tag{17}$$

where $\rho^D=r^D-\pi$.

Equation (17) implies that the present value of $PFB^{NL}_t$ relative to $D^{GG}_{t-1}$ has nothing to do with how high excess returns are from GG's risky investment. It is even similar to Equation (8), and the major source of positive cash flows from gross liabilities is again $\mathrm{E}(T^{GG}-G^{GG})>0$.

Two differences exist between constant discounting and stochastic discounting. First, higher constant discount rates are applied in the latter as $\rho^D+\frac{[\mathrm{E}(r^m-r^D)]^2}{\mathrm{E}(r^m-r^D)^2}>\rho^D$. Second, if $\beta^{T-G}$ is negative, or $\frac{T^{GG}_t-G^{GG}_t}{D^{GG}_{t-1}}$ is negatively correlated with $r^m_t-r^D$, then the discounted cash flows from net liabilities improve. This consequence is that countercyclical $T^{GG}_t-G^{GG}_t$ has an insurance effect on the discounted cash flows.

Restating the above implication, GG's life-time budget constraint (17) is independent of how the GG allocates funds among various risky assets. What does matter directly for GG's budget constraint is not the future possibility of capital gains and losses, but the past realizations of capital gains and losses, which have already been reflected in the current valuation of the gross assets, and are deducted from the gross liabilities.

### 2.3. Case of the failure of the Euler equations with respect to excess returns

As demonstrated in the previous subsection, as a consequence of strong restrictions from asset pricing theory, realized and unrealized returns from risky investment $(r^{GG,R}_t+r^{GG,UR}_t)$

disappear in a transition from Equation (6) or (7) to Equation (8) under constant discounting, and from Equation (14) to Equation (15) or (17) under stochastic discounting. In principle, such investment in risky assets has no impact on the discounted PFB or its present value.

However, it is well known that the orthogonality condition between stochastic discount factors and excess returns, which is represented by the Euler Equation (12), often breaks down because of various transaction constraints. In particular, excess returns tend to be too high on average given a certain stochastic factor. Consequently, investing short in risk-free assets and long in risky assets opens arbitrage opportunities for investors who can hold such risky assets. In the current cases, the Euler relationship does not hold with equality, but inequality as follows:

$$\frac{1}{1+r^D}\mathrm{E}_{t-1+\tau}\left(r_{t+\tau}^{GG,R}+r_{t+\tau}^{GG,UR}-r^D\right)=c_t^{GG}>0,$$

and

$$\mathrm{E}_{t-1+\tau}\left[\frac{1-\frac{\mathrm{E}\left(r^m-r^D\right)}{\mathrm{E}\left(r^m-r^D\right)^2}\left(r_{t+\tau}^m-r^D\right)}{1+r^D}\frac{\left(r_{t+\tau}^{GG,R}+r_{t+\tau}^{GG,UR}-r_{t+\tau}^D\right)A_{t-1+\tau}^{GG}}{D_{t-1+\tau}^{GG}}\right]=c_t^{GG}>0.$$

Suppose that Equation (11) holds by construction, but Equation (12) fails to hold because of some transaction constraints as follows:

$$\mathrm{E}_{t-1}\left[\frac{1-\frac{\mathrm{E}\left(r^m-r^D\right)}{\mathrm{E}\left(r^m-r^D\right)^2}\left(r_t^m-r^D\right)}{1+r^D}\left(r_t^i-r^D\right)\right]=c^i>0. \tag{18}$$

Note that $c^i$ is constant over time as a simplifying assumption.

Together with Equation (10), $\beta$-CAPM is modified from Equation (13) to that with a positive constant term $\alpha^i$:

$$r_t^i-r^D=\alpha^i+\beta^i\left(r_t^m-r^D\right)+\eta_t^i, \tag{19}$$

where $\alpha^i=c^iE[r^m-r^D]>0$,[6] and $\beta^i=\frac{Cov\left(r_t^i-r^D,r_t^m-r^D\right)}{Var\left(r_t^m-r^D\right)}$.

[6] Note that $\alpha^i$ in Equation (19) is different from $\alpha^i=\frac{\mathrm{Var}_t\left(r_t^i-r^d\right)}{\mathrm{E}_t\left(r_t^i-r^d\right)}$ in Chien et al. (2025).

If $\alpha^i$ in Equation (19) is significantly positive in the $\beta$-CAPM regression, then investment in asset $i$ creates arbitrage opportunities for those who hold long positions in this asset. For example, borrowing at $r^D$, investing loaned money in asset $i$, and immediately selling it in the futures market results in a budget-free profit for them. Note that such positive $\alpha^i$ is reflected in a higher $\overline{\beta}^i$ ($> \beta^i$) in the $\beta$-CAPM regression without a constant term.

Using Equations (16) and (19), $\frac{(T^{GG}_{t+\tau}-G^{GG}_{t+\tau})+\left(r^{GG,R}_{t+\tau}+r^{GG,UR}_{t+\tau}-r^{D}_{t+\tau}\right)A^{GG}_{t-1+\tau}}{D^{GG}_{t-1+\tau}}$ can be replaced by $\frac{\left[\alpha^{T-G}+\beta^{T-G}(r^m_{t+\tau}-r^D)+\eta^{T-G}_t\right]+\left[\alpha^{GG}+\beta^{GG}(r^m_{t+\tau}-r^D)+\eta^{GG}_{t+\tau}\right]A^{GG}_{t-1+\tau}}{D^{GG}_{t-1+\tau}}$. We discount this time $t+\tau$ payoff as of time $t-1+\tau$ by two types of discount factors. First, applying the same stochastic factor implicit in $\beta$-CAPM, we obtain:

$$\mathrm{E}_{t+\tau-1}\left\{\frac{1-\frac{\mathrm{E}\left(r^m-r^D\right)}{\mathrm{E}\left(r^m-r^D\right)^2}(r^m_t-r^D)}{1+r^D}\left[\left[\alpha^{T-G}+\beta^{T-G}(r^m_{t+\tau}-r^D)+\eta^{T-G}_t\right]+\frac{\left[\alpha^{GG}+\beta^{GG}(r^m_{t+\tau}-r^D)+\eta^{GG}_{t+\tau}\right]A^{GG}_{t-1+\tau}}{D^{GG}_{t-1+\tau}}\right]\right\}$$

$$=\frac{1-\frac{\left[\mathrm{E}\left(r^m-r^D\right)\right]^2}{\mathrm{E}\left(r^m-r^D\right)^2}}{1+r^D}\left[\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D^{GG}_{-1}}\right)-\beta^{T-G}E(r^m-r^D)+\alpha^{GG}\frac{A^{GG}_{t-1+\tau}}{D^{GG}_{t-1+\tau}}\right]. \tag{20}$$

Note that $r^m_{t+\tau}-r^D$ is orthogonal to $\left[1-\frac{\mathrm{E}(r^m-r^D)}{\mathrm{E}(r^m-r^D)^2}(r^m_t-r^D)\right]/(1+r^D)$ on the average. In Equation (20), $\beta^{GG}$ again disappears, but $\alpha^{GG}>0$ shows up.

Second, we adopt a constant discount factor as a case of the most drastic violation of the orthogonality condition. Thus, we derive:

$$\frac{1}{1+\rho^D}\mathrm{E}_{t+\tau-1}\left[\frac{(T^{GG}_{t+\tau}-G^{GG}_{t+\tau})+(\alpha^{GG}+\beta^{GG}(r^m_{t+\tau}-r^D)+\eta^{GG}_{t+\tau})A^{GG}_{t-1+\tau}}{D^{GG}_{t-1+\tau}}\right]$$

$$=\frac{1}{1+\rho}\left[\left(\alpha^{T-G}+\beta^{T-G}\mathrm{E}(r^m-r^D)\right)+\left(\alpha^{GG}+\beta^{GG}\mathrm{E}(r^m-r^D)\right)\frac{A^{GG}_{t-1+\tau}}{D^{GG}_{t-1+\tau}}\right], \tag{21}$$

where $\frac{1}{1+\rho}\left(\alpha^{GG}+\beta^{GG}\mathrm{E}(r^m-r^D)\right)\frac{A^{GG}_{t-1+\tau}}{D^{GG}_{t-1+\tau}}>0$, and $\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D^{GG}_{-1}}\right)=\alpha^{T-G}+\beta^{T-G}\mathrm{E}(r^m-r^D)$. The second term of the right-hand side of Equation (21) ($\frac{1}{1+\rho}\left(\alpha^{GG}+\beta^{GG}\mathrm{E}(r^m-r^D)\right)\frac{A^{GG}_{t-1+\tau}}{D^{GG}_{t-1+\tau}}$) never degenerates to zero, and it is instead positive in the presence of arbitrage opportunities for the government.

In sum, all $\alpha$s and $\beta$s of Equations (16) and (19) appear in Equation (21). That is, all parameters of the $\beta$-CAPM regression appear for not only the expected PFB, but also the discounted PFB. Applying $\beta$-CAPM to evaluate the expected PFB and applying constant factors to discount it are incompatible with each other. Thus, the second method is rather ad-hoc. Nevertheless, we still consider it because Equation (21) reflects the impact of the parameters of the $\beta$-CAPM regression to the fullest.

In the first case, the present value of the relative PFB is modified from Equation (17) to:

$$1-\mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right)\approx\frac{1}{\rho^{D}+\frac{\left[\mathrm{E}\left(r^{m}-r^{D}\right)\right]^{2}}{\mathrm{E}\left(r^{m}-r^{D}\right)^{2}}}\left[\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)-\beta^{T-G}E\left(r^{m}-r^{D}\right)+\alpha^{GG}\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right]. \tag{22}$$

As pointed out previously, $\overline{\beta}^{GG}>\beta^{GG}$ where $\overline{\beta}^{GG}$ corresponds to the $\beta$-CAPM regression without a constant term. Thus, $\alpha^{GG}\frac{A_{-1}^{GG}}{D_{-1}^{GG}}$ may be approximated by $\left(\overline{\beta}^{GG}-\beta^{GG}\right)\mathrm{E}\left(r^{m}-r^{D}\right)\mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right)>0$.

In the second case, however, it modified from Equation (8) to:

$$1-\mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right)=\frac{1}{\rho^{D}}\left[\left(\alpha^{T-G}+\beta^{T-G}\mathrm{E}\left(r^{m}-r^{D}\right)\right)+\left(\alpha^{GG}+\beta^{GG}\mathrm{E}\left(r^{m}-r^{D}\right)\right)\mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right)\right]. \tag{23}$$

Unlike in Equations (17) and (8), some or all parameters from the $\beta$-CAPM regression appear in Equations (22) and (23). That is, GG's behavior in risky investment finally appears in the evaluation of the present value of the future PFB. In Equation (22), how the GG exploits arbitrage opportunities with positive $\alpha^{GG}$ improves the present value of the PFB. In Equation (23), though it is quite ad-hoc, how the GG holds risky assets with high $\beta^{GG}$ enhances it.

In Equation (23), the effects of the $\alpha$s and $\beta$s on the discounted PFB is equivalent to those on the expected PFB. In this regard, Equation (23) is important in comparing between Chien et al. and our paper. Chien et al. consider possible effects from the $\beta$-CAPM regression at the level of, not the discounted PFB, but the expected PFB.

More concretely, all $\alpha$s are set equal to zero in Chien et al. Our $\beta^{T-G}$ corresponds to their $\frac{D_t-A_t}{D_t}\beta_t^S$, where $\beta_t^S$ is the $\beta$ of the conventional surplus $\frac{T_t-G_t}{D_t-A_t}$, while our $\frac{A_t}{D_t}\beta^{GG}$ corresponds to their $\frac{A_t}{D_t}\beta_t^A$. According to their calibration (not estimation), both the $\beta$s of the surplus claim and the risky assets are quite high. Given $\frac{A_t}{D_t}=0.66$, $\beta_t^S=0.45$, and $\beta_t^A=0.5$, $\beta$ of the gross liability is equal to $(1-0.66)\times 0.45+0.66\times 0.5=0.48$. One of our empirical goals is to compare this number 0.48 with our estimate of $\beta^{T-G}\mathrm{E}\left(r^{m}-r^{D}\right)+\frac{A^{GG}}{D^{GG}}\beta^{GG}\mathrm{E}\left(r^{m}-r^{D}\right)$ in Equation (23).

In the next section, we verify the statement by Chien et al. (2025) in two empirical respects. First, we explore whether arbitrage opportunities are really available to the Japanese government. Second, granting that arbitrage opportunities are present for the government, we examine whether the estimated $\beta$s of its net position are indeed high. In particular, we investigate whether the degree to which the government is making risky investment contributes to a change in the discounted PFB from negative to positive.

## 3. Construction of a net liability version of the PFB from the Flow of Funds Accounts

### 3.1. Data sources

To compute the PFB, we primarily use the quarterly Flow of Funds Accounts, compiled by the BoJ. These accounts consist of stock tables (financial assets and liabilities), flow tables (financial transactions), and reconciliation tables (reconciliation between flows and stocks) from the first quarter of 1998 to the fourth quarter of 2024. In addition, we obtain the data series of interest paid by the GG from the annual report of the National Accounts[7] from the first quarter of 1998 to the first quarter of 2024.

An increment in the net liabilities of the GG ($\Delta(D_t^{GG} - A_t^{GG})$) can be computed by the first-difference between $D_t^{GG} - A_t^{GG}$ and $D_{t-1}^{GG} - A_{t-1}^{GG}$ from the stock table. Out of $\Delta(D_t^{GG} - A_t^{GG})$, the realized components ($\Delta(D_t^{GG} - A_t^{GG})^R$) correspond to the financial surplus/deficit of the flow table, while the unrealized components ($\Delta(D_t^{GG} - A_t^{GG})^{UR}$) correspond to that of the reconciliation table. Then, $\Delta(D_t^{GG} - A_t^{GG}) = \Delta(D_t^{GG} - A_t^{GG})^R + \Delta(D_t^{GG} - A_t^{GG})^{UR}$ holds.

There are clear seasonal patterns in the realized components ($\Delta(D_t^{GG} - A_t^{GG})^R$). Thus, we take one-year moving averages of $(\Delta(D_t - A_t)^R$ for not only the GG, but also the central and local governments (denoted by CLGs) and the social security funds (denoted by SSFs). Thanks to adopting one-year moving averages instead of seasonal adjustment, $\Delta(D_t^{GG} - A_t^{GG})^R = \Delta(D_t^{CLGs} - A_t^{CLGs})^R + \Delta(D_t^{SSFs} - A_t^{SSFs})^R$ holds exactly, but the series starts from not the first quarter of 1998, but its fourth quarter, and still ends at the fourth quarter of 2024. Accordingly, the sample period is from the fourth quarter of 1998 to the fourth quarter of 2024, which is almost the same as that of Chien et al.'s calibration.

The series of interest paid by the GG ($r_r^D D_t^{GG}$) ends at the first quarter of 2024. Thus, the same numbers as the second quarter through the fourth quarter of 2023 are inserted into the corresponding quarters of 2024. Because there are also seasonal patterns in $r_r^D D_t^{GG}$, a one-year moving average is taken for the series. The sample period is then the same as above. The

[7] See Economic and Social Research (2024).

opportunity cost of holding gross assets $A_{t-1}^{GG}$ is computed by $r_t^D A_{t-1}^{GG} = (r_r^D D_t^{GG})\frac{A_{t-1}^{GG}}{D_{t-1}^{GG}}$, where $\frac{A_{t-1}^{GG}}{D_{t-1}^{GG}}$ is available from the stock table. Similarly, $r_t^D A_{t-1}^{CLGs} = (r_r^D D_t^{GG})\frac{A_{t-1}^{CLGs}}{D_{t-1}^{GG}}$, and $r_t^D A_{t-1}^{SSFs} = (r_r^D D_t^{GG})\frac{A_{t-1}^{SSFs}}{D_{t-1}^{GG}}$.

### 3.2. Observed values of several versions of the PFB

We can now compute gross and net liability versions of the PFB. From Equation (1), we derive a gross liability version of the PFB:

$$PFB_t^{GG,gross} = (T_t^{GG} - G_t^{GG}) + r_t^{GG,R} A_{t-1}^{GG} - \Delta A_t^{GG,R} = -\Delta D_t^{GG} + r_t^D D_{t-1}^{GG}, \qquad (1')$$

where $\Delta D_t^{GG}$ can be computed from the stock table, and $r_t^D D_{t-1}^{GG}$ is available from the National Accounts.

From Equation (3), we derive a net liability version of the PFB:

$$PFB_t^{GG,net} = (T_t^{GG} - G_t^{GG}) + \left(r_t^{GG,R} + r_t^{GG,UR} - r_t^D\right) A_{t-1}^{GG} = -\Delta(D_t^{GG} - A_t^{GG}) + r_t^D D_{t-1}^{GG} - r_t^D A_{t-1}^{GG}. \qquad (3')$$

Equation (3') can be decomposed into realized and unrealized components:

$$PFB_t^{GG,net,R} = (T_t^{GG} - G_t^{GG}) + r_t^{GG,R} A_{t-1}^{GG} = -\Delta(D_t^{GG} - A_t^{GG})^R + r_t^D D_{t-1}^{GG}, \qquad (3'\text{-}1)$$

$$PFB_t^{GG,net,UR} = \left(r_t^{GG,UR} - r_t^D\right) A_{t-1}^{GG} = -\Delta(D_t^{GG} - A_t^{GG})^{UR} - r_t^D A_{t-1}^{GG}. \qquad (3'\text{-}2)$$

Equation (3'-2) is further divided into CLGs' assets and SSFs' assets:

$$PFB_t^{CLGs,UR} = \left(r_t^{CLGs,UR} - r_t^D\right) A_{t-1}^{CLGs} = -\Delta(D_t^{CLGs} - A_t^{CLGs})^{UR} - r_t^D A_{t-1}^{CLGs}, \qquad (3'\text{-}2\text{-}1)$$

$$PFB_t^{SSFs,UR} = \left(r_t^{SSFs,UR} - r_t^D\right) A_{t-1}^{SSFs} = -\Delta(D_t^{SSFs} - A_t^{SSFs})^{UR} - r_t^D A_{t-1}^{SSFs}. \qquad (3'\text{-}2\text{-}2)$$

The above equations represent unrealized excess returns for CLGs and SSFs.

In addition, realized and unrealized net returns from net assets for the BoJ as another body of the IG is determined as follows:

$$R_t^{BoJ,net} = \Delta\left(A_t^{BoJ} - D_t^{BoJ}\right) = \Delta\left(A_t^{BoJ} - D_t^{BoJ}\right)^R + \Delta\left(A_t^{BoJ} - D_t^{BoJ}\right)^{UR}. \qquad (24)$$

The BoJ never paid any interest on required and excess reserves up to October 2008. From then, it started to pay interest rates equal to only 0.1% or less on excess reserves. The BoJ's gross liabilities occupy around 95% of its gross assets. Thus, $r_t^{D,BoJ} D_{t-1}^{BoJ} - r_t^{D,BoJ} A_{t-1}^{BoJ}$ is judgeable to be quite small, and the BoJ's net returns on net assets are almost equivalent to its PFB:

$$PFB_t^{BoJ,net} = R_t^{BoJ,net} + r_t^{D,BoJ} D_{t-1}^{BoJ} - r_t^{D,BoJ} A_{t-1}^{BoJ} \approx R_t^{BoJ,net}.$$

Figure 3 depicts the time series of a net liability version of the PFB without unrealized returns for the GG ($PFB_t^{GG,net,R}$, a red solid line), its net liability version with them ($PFB_t^{GG,net}$, a black solid line), and its gross liability version ($PFB_t^{GG,gross}$, a red dotted line).

(insert Figure 3)

The series of $PFB_t^{GG,net,R}$ and $PFB_t^{GG,gross}$, either of which does not include any unrealized returns from gross assets, are quite close to each other. Both series are chronically negative for the entire sample period. The only difference between the two is that the latter is lower by net asset purchases if $\Delta A_t^{GG,R} > 0$.

However, once unrealized returns from gross assets are included in the PFB, the series become quite volatile, and often positive. In particular, the series of $PFB_t^{GG,net}$ frequently record large positive numbers from the mid-2010s. Adjusted by one-period lagged gross liabilities $D_{t-1}^{GG}$ and converted into annual rates, the full-sample averages of $\frac{PFB_t^{GG,net,R}}{D_{t-1}^{GG}}$, $\frac{PFB_t^{GG,net}}{D_{t-1}^{GG}}$, and $\frac{PFB_t^{GG,gross}}{D_{t-1}^{GG}}$ are computed as −2.1% per quarter with standard deviation 0.8%, −1.4% with 2.9%, and −3.5% with 1.4%, respectively.

We have two remarks on these sample averages. First, the full-sample average of $\frac{PFB_t^{GG,net}}{D_{t-1}^{GG}}$ is still negative even with unrealized returns included. Second, the sample average of $\frac{PFB_t^{GG,net,R}}{D_{t-1}^{GG}}$ serves as the upper bound for the average of the relative conventional PFB appearing in Equations (8) and (17), or $\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)$. Thus, the theoretically consistent present value of the future PFB is largely negative.

As a byproduct of Equation (1), we can compute the series of quarterly interest rates on

GG's liabilities ($r_t^D$) given $D_{t-1}^{GG}$. As shown in Figure 4, the borrowing rate for GG declined over time. The quarterly rate was around 0.7% in the late 1990s, but it decreased to 0.15% in the early 2020s.

(insert Figure 4)

We demonstrate below that our computed PFBs are comparable to those of Chien et al. (2025). Chien et al. split their full sample of 1998–2023 into subsamples of 1998–2012 and 2013–2013, while we divide that of 1999–2024 into 1999–2012 and 2013–2024. As the area shaded by blue in Table 1 shows, net returns on gross assets held by the IG (GG plus BoJ) are computed with interest on government bonds added as realized returns in our computation. According to the two panels of Table 1, our total returns are 2.08% for the full sample, while their net returns are 2.28%. Similarly, 0.55% is relative to −0.04% for the first subsample, and 3.93% to 4.66% for the second subsample.

In addition, as the area shaded by yellow in Table 1 shows, the magnitude of unrealized returns on gross assets held by the IG, relative to nominal gross domestic product, in our computation is also comparable to the relative magnitude of net returns on gross assets in theirs. For the full sample, the relative magnitude is 1.75% in ours, while it is 2.28% in theirs. For the first (second) subsample, it is −0.37% (4.34%) in the former, while it is −0.63% (6.25%) in the latter.

### 3.3. Interpreting the observed PFB in terms of the $\boldsymbol{\beta}$-CAPM regression

Let us interpret the observed PFB in terms of the $\beta$-CAPM regression. Excess market returns, $r_t^m - r_t^D$, are constructed as follows. Market returns from Nikkei 225 are computed on a quarter-end to quarter-end basis. Quarterly dividend returns from the first section, or the prime section of the Tokyo Stock Exchange are added to market returns, while quarterly yields on one-year JGBs are subtracted from them. The full sample average of $r_t^m - r_t^D$ is 8.1% at annual rates with a standard deviation of 20.4%. Thus, the Sharpe ratio is 0.40:

Both $\frac{PFB_t^{GG,net,R}}{D_{t-1}^{GG}}$ and $\frac{PFB_t^{GG,net}}{D_{t-1}^{GG}}$ are regressed on $r_t^m - r_t^D$. $\beta^{GG,R}$ and $\beta^{GG,UR}$ are the estimated coefficient on $r_t^m - r_t^D$, while $\alpha^{GG,R}$ and $\alpha^{GG,UR}$ are the estimated constant term. Table 2 reports the estimation results for not only the full sample, but also the subsamples 1999–2012 and 2013–2024. Figures 5-1 and 5-2 present a scatter diagram of excess market returns on the X axis, and $\frac{PFB_t^{GG,net,R}}{D_{t-1}^{GG}}$ or $\frac{PFB_t^{GG,net}}{D_{t-1}^{GG}}$ on the Y axis for the full sample.

(insert Table 2)

(insert Figure 5-1)
(insert Figure 5-2)

As $\beta^{GG,R} < 0$ implies, $\frac{PFB_t^{GG,net,R}}{D_{t-1}^{GG}}$ is negatively, though insignificantly, correlated with $r_t^m - r_t^D$. This small negative correlation implies less cyclical tax revenues $T_t^{GG}$. The significantly negative constant ($\alpha^{GG,R} < 0$), however, suggests that $T_t^{GG} - G_t^{GG}$ is largely negative on average. The pattern of significantly negative $\alpha$ and slightly negative $\beta$ does not change between the subsamples 1999–2012 and 2013–2024.

Once unrealized returns are added to the PFB, estimated $\beta^{GG}$ changes from weakly negative to significantly positive, while estimated $\alpha^{GG}$ does not change that much, and remains negative. That is, the average PFB improves to the extent that the GG takes market risks. For an average quarterly excess market return of 2.0%, the average relative PFB improves from −0.52% to −0.36%, but remains negative. The pattern of significantly negative $\alpha$ and significantly positive $\beta$ does not change between the two subsamples. But $\beta$ increases from 0.053 in the years 1999–2012 to 0.101 in 2013–2024. Accordingly, the expected relative PFB changes from negative (−0.8%) in the years 1999–2012 to positive (0.15%) in 2013–2024 at the corresponding average excess market return (0.4% and 3.8% respectively).

The above estimation results are dramatically different from Chien et al.'s calibration results. Let us take the full-sample results as an example. As cited in Section 2.3, they set all $\alpha$s at zero. In our estimation, both $\alpha^{CLG,UR}$ and $\alpha^{SSF,UR}$ are also close to zero. But $\alpha^{GG,R}$ is significantly negative, while $\alpha^{BoJ,net}$ is positive though insignificant. However, they calibrate $\beta^A$ ($\beta$ of net returns on gross assets) to be 0.5. Such $\beta$ is much higher than our estimates $\beta^{CLG,UR} = 0.14$ ($\beta$ of net returns on gross assets held by the central and local governments), $\beta^{SSF,UR} = 0.18$ (by the social security funds), or $\beta^{BoJ,net} = 0.10$ (on net assets held by the BoJ). Their positive $\beta$ of the surplus claim (0.15) suggests that the primary surplus is highly cyclical, while our $\beta^{T-G} = -0.005$ implies that it is slightly countercyclical. Finally, their high $\beta$ of net returns on gross liabilities carried by the IG (0.48) differs sharply from $\beta^{GG,net} = 0.077$ ($\beta$ of net returns on gross assets held by the GG).

### 3.4. Were arbitrage opportunities available to the government?

As demonstrated in Section 2-3, a high $\beta$ affects not only the expected PFB, but also the discounted PFB only if arbitrage opportunities are available to the government. We now explore whether arbitrage opportunities were indeed present for the government using two simple methods.

First, we adopt Sharpe ratios for net returns on gross assets held by the government bodies, which are reported by Table 3. In the full sample, the Sharpe ratios for the central and local governments (0.29), social security funds (0.39), and the BoJ's net positions (0.35) are all dominated by that of excess market returns (0.40). While the overall Sharpe ratios improved remarkably in the years 2013–2024, the Sharpe ratios of the three bodies (0.45, 0.64, and 0.50) are still dominated by that of excess market returns (0.86). As far as the Sharpe ratios are concerned, it is difficult to conclude that the IG could hold market-dominating and effective portfolios.

Second, as shown in Section 2-3, a positive constant term in the $\beta$-CAPM regression ($\alpha > 0$) implies that arbitrage opportunities may be present for those holding assets in concern. Table 2 reports the estimation results of the $\beta$-CAPM regression for unrealized net returns on gross assets held by the above three government bodies in both the full samples and the two subsamples. Figures 6-1 to 6-3 scatter unrealized net returns on gross assets against excess market returns in the full sample.

According to these tables and figures, the estimated $\beta$s are positive for the three bodies in the full sample and the two subsamples, and they are significant except for the BoJ's net position in the first subsample. However, the pattern in estimated $\beta$ differs between the years 1999–2012 and 2013–2024. That is, $\beta^{CLG,UR}$ is around 0.3 between the two subsamples, but $\beta^{SSW,UR}$ increases substantially from 0.089 in 1999–2012 to 0.320 in 2013–2025. $\beta^{BoJ,R+UR}$ changes from negative ($-0.229$) to positive (0.613).

(insert Figure 6-1)
(insert Figure 6-2)
(insert Figure 6-3)

In the full sample, however, the estimated constant terms ($\alpha$) are small in magnitude for the central and local governments and the social security funds. In addition, they are not significantly different from zero. The estimation results do not change between the years 1999–2012 and 2013–2024. More concretely, $\alpha^{CLG,UR}$ is around 0.3%, and $\alpha^{SSW,UR}$ is between $-0.2\%$ and 0.1%. For the BoJ's net positions, estimated $\alpha$ is relatively large in magnitude, but remains insignificant. That is, $\alpha^{BoJ,R+UR}$ is between 1.4% and 2.9%. In any case, estimated $\alpha$ is by no means significantly positive.

As demonstrated above, the estimation results of the $\beta$-CAPM regression fail to reject the absence of arbitrage opportunities for the government, in which the null hypothesis is $\alpha = 0$. Thus, it is again difficult to conclude that effective arbitrage opportunities are present for the government, and that the high $\beta$s of the financial and fiscal positions are immediately translated to the discounted PFB.

### 3.5. Computing $\alpha$ and $\beta$ of the IG's PFB

As shown in the previous subsection, we cannot find any strong evidence for the presence of arbitrage opportunities for the government. Nevertheless, our evidence for its absence is rather weak. Thus, granting that arbitrage opportunities are available to the government, we next consider whether the high $\beta$ can improve not only the expected PFB, but also the discounted PFB. More concretely, we explore whether the estimated $\beta$ is so high that the discounted PFB can change from negative to positive. In particular, we carefully examine whether the positive expected PFB on gross liabilities indeed emerged from a high $\beta$ in the years 2013–2024, which would constitute major empirical support for the statement by Chien et al.

We compute $\alpha$ and $\beta$ for the IG, consisting of the GG and the BoJ for the full sample under simplifying assumptions. The PFB relative to gross liabilities for the IG ($\frac{PFB_t^{IG,net}}{D_{t-1}^{GG}}$) is decomposed as:

$$\frac{PFB_t^{IG,net}}{D_{t-1}^{GG}} = \frac{PFB_t^{GG,net,R}}{D_{t-1}^{GG}} + \frac{PFB_t^{CLGs,UR}}{A_{t-1}^{CLGs}} \frac{A_{t-1}^{CLGs}}{D_{t-1}^{GG}} + \frac{PFB_t^{CLGs,UR}}{A_{t-1}^{SSFs}} \frac{A_{t-1}^{SSFs}}{D_{t-1}^{GG}} + \frac{R_t^{BoJ,net}}{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}} \frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}.$$

The above equation is heroically approximated as:

$$\frac{PFB_t^{IG,net}}{D_{t-1}^{GG}} \approx \frac{PFB_t^{GG,net,R}}{D_{t-1}^{GG}} + \frac{PFB_t^{CLGs,UR}}{A_{t-1}^{CLGs}} \mathrm{E}\left(\frac{A_{t-1}^{CLGs}}{D_{t-1}^{GG}}\right) + \frac{PFB_t^{CLGs,UR}}{A_{t-1}^{SSFs}} \mathrm{E}\left(\frac{A_{t-1}^{SSFs}}{D_{t-1}^{GG}}\right) + \frac{R_t^{BoJ,net}}{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}} \mathrm{E}\left(\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}\right). \quad (25)$$

If excess returns ($r_t^{CLGs,UR} - r_t^D$, $r_t^{SSFs,UR} - r_t^D$, and $\frac{R_t^{BoJ,net}}{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}$) are uncorrelated with the asset share ($\frac{A_{t-1}^{CLGs}}{D_{t-1}^{GG}}$, $\frac{A_{t-1}^{SSFs}}{D_{t-1}^{GG}}$, and $\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}$) as assumed in Sections 2-1 and 2-2, Equation (25) is an exact representation. As shown in Figure 7, however, $\frac{A_{t-1}^{CLGs}}{D_{t-1}^{GG}}$, $\frac{A_{t-1}^{SSFs}}{D_{t-1}^{GG}}$, and $\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}$ increased as asset prices appreciated from the mid-2010s, and Equation (19) is accordingly a bold approximation. Nevertheless, we assign below the full-sample averages of 0.300 to $\mathrm{E}\left(\frac{A_{t-1}^{CLGs}}{D_{t-1}^{GG}}\right)$, 0.252 to $\mathrm{E}\left(\frac{A_{t-1}^{SSFs}}{D_{t-1}^{GG}}\right)$, and 0.015 to $\mathrm{E}\left(\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}\right)$, respectively, three of which appear on the right-hand side of Equation (25).

(insert Figure 7)

Given approximation (25), $\alpha^{IG}$ and $\beta^{IG}$ are expressed as:

$$\alpha^{IG} = \alpha^{GG,R} + \alpha^{CLG,UR}\mathrm{E}\left(\frac{A_{t-1}^{CLG}}{D_{t-1}^{GG}}\right) + \alpha^{SSF,UR}\mathrm{E}\left(\frac{A_{t-1}^{SSF}}{D_{t-1}^{GG}}\right) + \alpha^{BoJ,net}\mathrm{E}\left(\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}\right) \quad (26)$$

$$\beta^{IG} = \beta^{GG,R} + \beta^{CLG,UR}\mathrm{E}\left(\frac{A_{t-1}^{CLG}}{D_{t-1}^{GG}}\right) + \beta^{SSF,UR}\mathrm{E}\left(\frac{A_{t-1}^{SSF}}{D_{t-1}^{GG}}\right) + \beta^{BoJ,net}\mathrm{E}\left(\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}\right). \quad (27)$$

According to Equation (22), only the $\alpha$s on the right-hand side of Equation (26) are significant in the evaluation of the discounted PFB, while as Equation (23) implies, not only the $\alpha$s in Equation (26), but also the $\beta$s in Equation (27) matter. Given that the estimated $\beta^{T-G}$ and $\alpha$s ($\alpha^{CLG,UR}$, $\alpha^{SSF,UR}$, and $\alpha^{BoJ,net}\mathrm{E}\left(\frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}\right)$) are rather small as reported in Table 2, $\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right) - \beta^{T-G}E(r^m - r^D) + \alpha^{GG}\frac{A_{-1}^{GG}}{D_{-1}^{GG}} \approx \mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right)$, and the estimated constant terms of the $\beta$-CAPM regression have almost no impact on the discounted PFB in Equation (22). Thus, below we follow the interpretation of Equation (23), although the combination of $\beta$-CAPM and constant discounting is quite ad-hoc.

As demonstrated by Table 4, substituting the estimated $\alpha$s and $\beta$s from Table 2, and the full-sample averages into the right-hand side of Equations (26) and (27), $\alpha^{IG}$ is computed as $-0.0036$, and $\beta^{IG}$ as 0.0827. $\alpha^{IG}$ is close to $\alpha^{GG,net}$ ($-0.0051$), while $\beta^{IG}$ is a little higher than $\beta^{GG,net}$ (0.0770). Figure 8 draws four linear functions with (A) a pair of $\alpha^{GG,R}$ and $\beta^{GG,R}$, (B) a pair of $\alpha^{GG,R} + 0.3\alpha^{CLG,UR}$ and $\beta^{GG,R} + 0.3\beta^{CLG,UR}$, (C) a pair of $\alpha^{GG,R} + 0.3\alpha^{CLG,UR} + 0.252\alpha^{SSF,UR}$ and $\beta^{GG,R} + 0.3\beta^{CLG,UR} + 0.252\beta^{SSF,UR}$, and (D) a pair of $\alpha^{GG,R} + 0.3\alpha^{CLG,UR} + 0.252\alpha^{SSF,UR} + 0.015\alpha^{BoJ,net}$ and

$\beta^{GG,R} + 0.3\beta^{CLG,UR} + 0.252\beta^{SSF,UR} + 0.015\beta^{BoJ,net}$.

(insert Table 4)

(insert Figure 8)

Figure 8 illuminates how risky investment by the CLGs, the SSFs, and the BoJ affects the risk-return structure on the IG's gross liabilities. Without any risky investment, a linear line is slightly downward-sloping. Including risky investments one by one, however, the slope of linear lines becomes steeper and steeper from $-0.0051$ to 0.0356 by the CLGs' investment, to 0.0812 by the SSFs', and to 0.0827 by the BoJ's. The constant terms remain around $-0.004$.

If the four lines are evaluated using the full-sample quarterly average excess market return of 2%, the expected excess return on GI's gross liabilities improves from $-0.0052$ to $-0.0035$, $-0.0023$, and $-0.0019$, respectively. Each increment ranges between 0.0004 and 0.0007, and is rather small. Even with all risky investments included, the expected excess return remains negative. This means that the government never yields a positive cash flow on average, although all parameters of the $\beta$-CAPM regression for excess returns on risky assets held by the CLGs and the SSFs, and net assets held by the BoJ are considered to the fullest.

As shown in Section 3-3, the expected relative PFB with unrealized returns added improved from $-0.8\%$ in the years 1999–2012 to 0.15% in 2013–2024. However, such positive expected PFBs came from not a high $\beta$, but from high market returns. As reported in Table 2, estimated $\beta^{GG,net}$ increases from the first subsample to the second, but its estimated magnitude of 0.101 remains small. As demonstrated in Figure 9, the $\beta$-line becomes steeper from the first subsample (lower dotted line) to the second (upper dotted line). At the same time, the average excess market return increases remarkably from 0.4% to 3.8% per quarter. If it remains at the full-sample average (2.0%), the expected PFB is still negative even if $\beta^{GG,net} = 0.101$. That is, in the years 2013–2024, a positive expected PFB emerged not from a high $\beta$, but from high market returns.

As discussed above, the discounted PFB bears at most a moderate $\beta$, even if all parameters of the $\beta$-CAPM regression are considered to the fullest under the assumption that arbitrage opportunities are available to the government. Thus, the present value of the future $PFB_t^{IG,net}$ is judged to be smaller than the current valuation of the net liabilities. Unless the average market returns are exceptionally high, the right-hand side of the inequality below is likely to be negative:

$$1 - \mathrm{E}\left(\frac{A_{-1}^{GG}}{D_{-1}^{GG}}\right) > 0 > \frac{1}{\rho^D}\left[\alpha^{IG} + \beta^{IG}\mathrm{E}(r^m - r^D)\right].$$

Of course, even in this case, the current gross liabilities are still sustainable if the real growth rate exceeds the real rate of interest ($\rho^D$).

In sum, it is difficult to immediately conclude that the Japanese government can exploit effective arbitrage opportunities, and that it can back its huge gross liabilities by high $\beta$ assets and claims.

### 3.6. Should the Japanese government take much more market risk to make the expected PFB positive?

According to the exercises in Sections 3.3 and 3.4, the government can make the expected PFB positive only if it takes much more market risk. Suppose that $\mathrm{E}\left(\frac{T^{GG}-G^{GG}}{D_{-1}^{GG}}\right) = -0.005$, $\mathrm{E}\left(\frac{A_{t-1}^{CLG}}{D_{t-1}^{GG}} + \frac{A_{t-1}^{SSF}}{D_{t-1}^{GG}} + \frac{A_{t-1}^{BoJ}-D_{t-1}^{BoJ}}{D_{t-1}^{GG}}\right) = 0.6$, and $\mathrm{E}(r_t^m - r_t^D) = 0.02$ at quarterly rates. If the IG bodies hold highly risky portfolios with $\beta \geq 0.42$, which is five times as large as the current $\beta \approx 0.08$, and comparable to Chien et al.'s calibration of $\beta^A = 0.5$, then the expected PFB are positive. In Figure 8, the black dotted line represents such high $\beta$ investment by the IG.

Do the taxpayers desire such a dramatic change in the government's asset portfolios? Note that $\beta^{GG}$ appears in Equation (23) not because of stochastic discounting, but because constant discounting is applied. That is, the government is risk neutral, and it does not care about how volatile the government's portfolios are. Given the rather conservative asset portfolios held by Japanese households, it is difficult to imagine that the assumption employed in Equation (23) is consistent with taxpayers' risk-averse attitudes.

Of course, there are alternative discussions about hypothetical asymmetry between the government's risky portfolios and the households' conservative portfolios. As Chien et al. assume, the Japanese government may be facing effective arbitrage opportunities. Given arbitrage opportunities for the government, there may be two interpretations. First, the government and taxpayers are corporative with each other. Most households prefer risky portfolios to conservative ones, but they cannot hold them because of several constraints. Thus, the government holds risky assets on behalf of the taxpayers.

Second, the government and the taxpayers are hostile to each other. Again, most households are forced to hold conservative portfolios because of several constraints. On the opposite side, there emerge arbitrage opportunities for unconstrained investors. Many interest groups govern the government's portfolio management independently of taxpayers' interests. The government would hold risky assets at the sacrifice of the forced households, and on behalf of these interest groups. According to interpretations by Chien et al., younger and less financially sophisticated households are those who are forced to hold conservative portfolios, while older and financially sophisticated households are those who belong to interest groups.

## 4. Conclusion

Our theoretical and empirical investigation is summarized as follows. In the past decade, the Japanese government enjoyed enormous amounts of unrealized capital gains, which were reflected in the market valuation of its risky assets, and it successfully reduced its net liabilities. It is indeed this remarkable valuation improvement that Chien et al. pay serious attention to. But the discounted PFB or its present value is directly determined, not by the past realization of capital gains, but by the future possibility of capital gains as well as losses. Under stochastic discounting, positive (negative) excess returns are discounted more (less) heavily, and accordingly the discounted excess returns degenerate to zero. In principle, the $\beta$s of the surplus claim and the risky assets have no impact on the discounted PFB.

Under constant discounting in which both positive and negative excess returns are discounted equally, however, there is room for the $\beta$s to affect not only the expected PFB, but also the discounted PFB. However, the estimated $\beta$s of the risky assets held by the IG have limited impacts on the discounted PFB because the government is currently taking only a moderate level of $\beta$ for its risky investment. Thus, the current valuation of huge gross liabilities cannot be backed by such moderate $\beta$ financial assets. It must be sustained by the sequence of conventional primary surpluses, $T_t^{GG} - G_t^{GG}$, unless growth rates continue to exceed interest rates. As discussed in this paper, given the arbitrage opportunities present for the government, its active investment in risky assets may be justifiable to some extent. Nevertheless, if the government left the chronic deficit of $T_t^{GG} - G_t^{GG}$ alone, but aggressively pursued unreasonably high $\beta$ for a dramatic improvement of the expected PFB, then the taxpayers might be forced to bear heavy capital losses beyond their risk capacity.

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**Table 1: Comparison of major basic statistics between Chien et al. (2025) and ours (annual rates)**

Chien et al. (2025)

| | Payment on gross liabilities | Net returns on gross assets | net returns on gross assets/nominal GDP |
|---|---|---|---|
| 1998-2023 | 0.69% | 1.90% | 2.28% |
| 1998-2012 | 0.92% | -0.04% | -0.63% |
| 2013-2023 | 0.38% | 4.66% | 6.25% |

Saito (2025)

| | (1) Excess returns on gross assets (for central and local governments) | (2) Excess returns on gross assets (for social security funds) | (3) Net returns on **net** assets (for Bank of Japan) | (4) Total net (excess) returns on gross assets (for CLGs and SSFs), and net assets for BoJ |
|---|---|---|---|---|
| 1999-2024 | 2.24% | 1.85% | 13.03% | 2.08% |
| 1999-2012 | 1.19% | -0.04% | 10.76% | 0.55% |
| 2013-2024 | 3.50% | 4.16% | 15.77% | 3.93% |

| | (5) net version of PFB without unrealized gains and losses for GG/nominal GDP | (6) net version of PFB with unrealized gains and losses for GG and BoJ/nominal GDP | (6)-(5) |
|---|---|---|---|
| 1999-2024 | -3.75% | -2.00% | 1.75% |
| 1999-2012 | -4.58% | -4.95% | -0.37% |
| 2013-2024 | -2.75% | 1.59% | 4.34% |

(1), (2), and (4) include interest on government bonds as realized returns.

**Table 2: Estimation results of β-CAPM regression**

| | PFB without unrealized returns on gross liabilities for general government | | PFB with unrealized returns on gross liabilities for general government | | Unrealized excess returns on gross assets for central and local governments | | Unrealized excess returns on gross assets for social security funds | | Net returns on net assets for Bank of Japan | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\alpha^{GG,R}$ | $\beta^{GG,R}$ | $\alpha^{GG,net}$ | $\beta^{GG,net}$ | $\alpha^{CLG,UR}$ | $\beta^{CLG,UR}$ | $\alpha^{SSW,UR}$ | $\beta^{SSW,UR}$ | $\alpha^{BoJ,net}$ | $\beta^{BoJ,net}$ |
| 1999-2024 | -0.0051 | -0.0051 | -0.0051 | 0.0770 | 0.0029 | 0.1356 | 0.0011 | 0.1813 | 0.0291 | 0.1003 |
| | (0.0004) | (0.0039) | (0.0012) | (0.0117) | (0.0036) | (0.0345) | (0.0015) | (0.0150) | (0.0186) | (0.1802) |
| 1999-2012 | -0.0072 | -0.0108 | -0.0080 | 0.0533 | 0.0023 | 0.1344 | -0.0006 | 0.0892 | 0.0269 | -0.2293 |
| | (0.0005) | (0.0051) | (0.0014) | (0.0135) | (0.0047) | (0.0441) | (0.0008) | (0.0078) | (0.0279) | (0.2613) |
| 2013-2024 | -0.0027 | -0.0057 | -0.0023 | 0.1006 | 0.0036 | 0.1349 | -0.0018 | 0.3200 | 0.0142 | 0.6132 |
| | (0.0003) | (0.0033) | (0.0019) | (0.0195) | (0.0058) | (0.0584) | (0.0022) | (0.0220) | (0.0230) | (0.2321) |

**Table 3: Comarison of Sharpe ratios (annual rates)**

| | Excess returns on Nikkei 225 | | | Unrealized excess returns on gross assets for central and local governments | | | Unrealized excess returns on gross assets for social security funds | | | Net returns on gross assets for Bank of Japan | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | average | standard deviation | Shrpe ratio | average | standard deviation | Shrpe ratio | average | standard deviation | Shrpe ratio | average | standard deviation | Shrpe ratio |
| 1999-2024 | 8.09% | 20.35% | **0.40** | 2.24% | 7.65% | **0.29** | 1.86% | 4.81% | **0.39** | 13.03% | 37.27% | **0.35** |
| 1999-2012 | 1.79% | 21.49% | **0.08** | 1.18% | 7.60% | **0.16** | -0.07% | 2.29% | **-0.03** | 10.76% | 41.93% | **0.26** |
| 2013-2024 | 15.94% | 18.49% | **0.86** | 3.52% | 7.74% | **0.45** | 4.18% | 6.53% | **0.64** | 15.77% | 31.23% | **0.50** |

**Table 4: Impacts of $\beta$ of risky investment of CLGs, SSFs, and BoJ on $\beta$ of the integrated government**

| | Realized PFB on gross liabilities for GG | Unrealized excess returns on gross assets for CLGs | Unrealized excess returns on gross assets for SSFs | Realized and unrealized net returns on net assets for BoJ | Computed $\beta$ of the integrated government |
|---|---|---|---|---|---|
| (i) average of gross (net) assets share/total gross liabilities | 1.000 | 0.300 | 0.252 | 0.015 | |
| (ii) estimated $\alpha$ | -0.0051 | 0.0029 | 0.0011 | 0.0291 | sum of (iii) |
| (iii) estimated impact ((i) × (ii)) | **-0.0051** | **0.0009** | **0.0003** | **0.0004** | **-0.0036** |
| (iv) estimated $\beta$ | -0.0051 | 0.1356 | 0.1813 | 0.1003 | sum of (V) |
| (v) estimated impact ((i) × (iv)) | **-0.0051** | **0.0407** | **0.0456** | **0.0015** | **0.0827** |

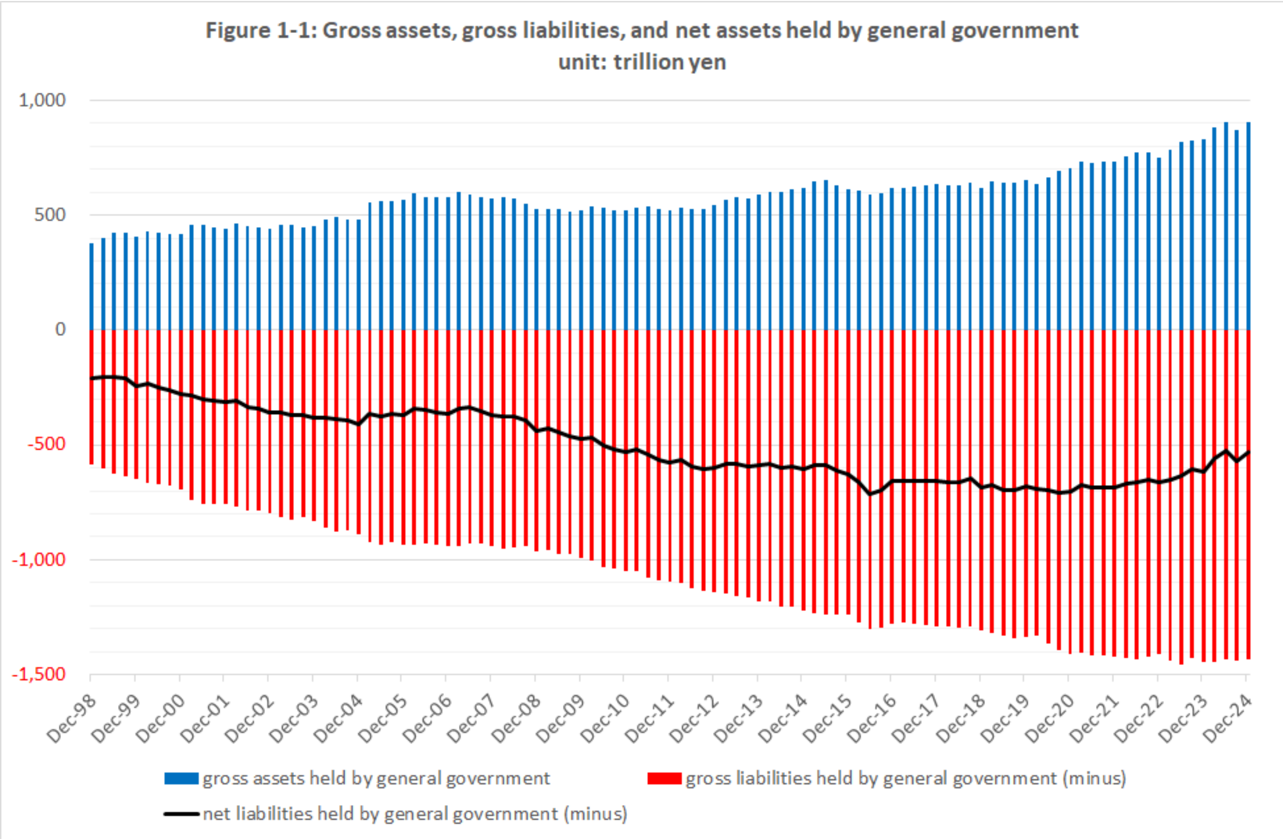


Figure 1-1: Gross assets, gross liabilities, and net assets held by general government
unit: trillion yen
1,000
500
0
-500
-1,000
-1,500
Dec-98
Dec-99
Dec-00
Dec-01
Dec-02
Dec-03
Dec-04
Dec-05
Dec-06
Dec-07
Dec-08
Dec-09
Dec-10
Dec-11
Dec-12
Dec-13
Dec-14
Dec-15
Dec-16
Dec-17
Dec-18
Dec-19
Dec-20
Dec-21
Dec-22
Dec-23
Dec-24
gross assets held by general government
gross liabilities held by general government (minus)
net liabilities held by general government (minus)

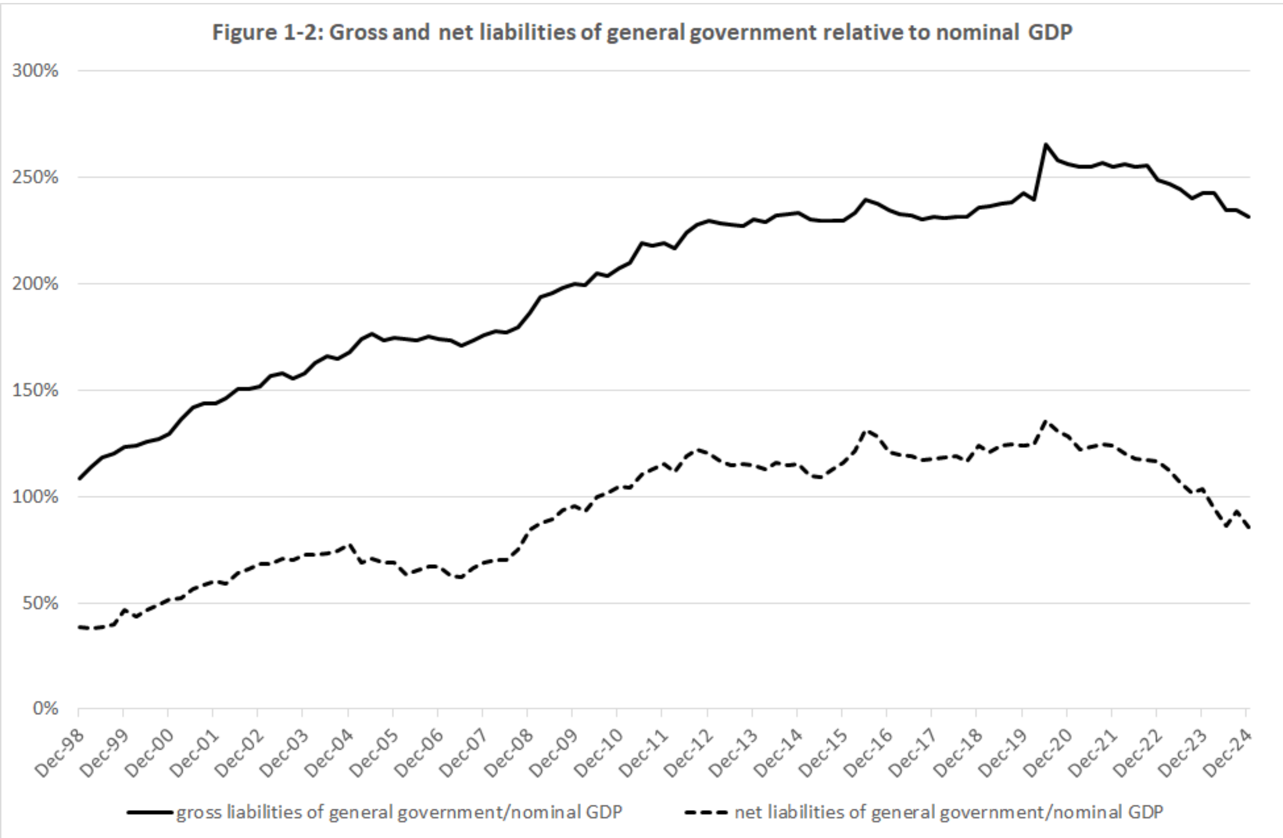


Figure 1-2: Gross and net liabilities of general government relative to nominal GDP
300%
250%
200%
150%
100%
50%
0%
Dec-98
Dec-99
Dec-00
Dec-01
Dec-02
Dec-03
Dec-04
Dec-05
Dec-06
Dec-07
Dec-08
Dec-09
Dec-10
Dec-11
Dec-12
Dec-13
Dec-14
Dec-15
Dec-16
Dec-17
Dec-18
Dec-19
Dec-20
Dec-21
Dec-22
Dec-23
Dec-24
gross liabilities of general government/nominal GDP
net liabilities of general government/nominal GDP

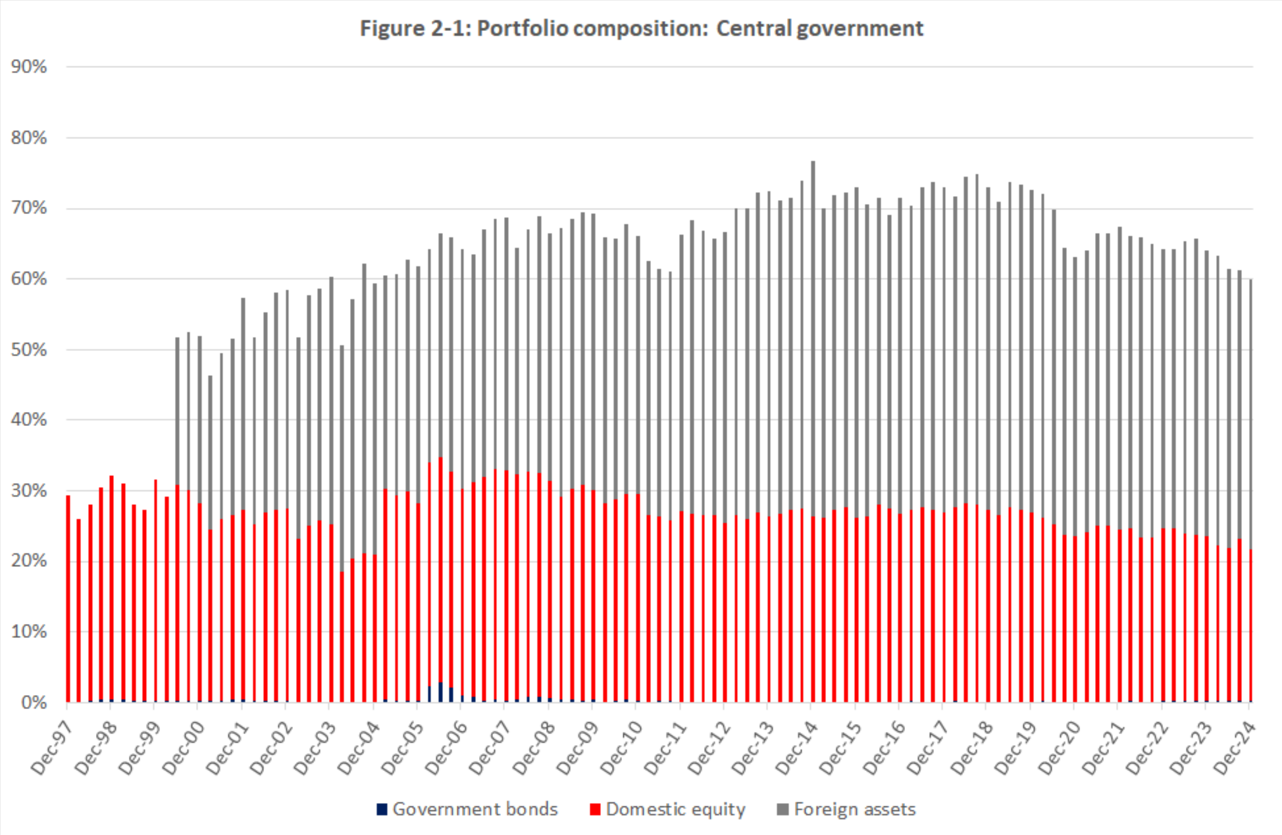


Figure 2-1: Portfolio composition: Central government
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Dec-97
Dec-98
Dec-99
Dec-00
Dec-01
Dec-02
Dec-03
Dec-04
Dec-05
Dec-06
Dec-07
Dec-08
Dec-09
Dec-10
Dec-11
Dec-12
Dec-13
Dec-14
Dec-15
Dec-16
Dec-17
Dec-18
Dec-19
Dec-20
Dec-21
Dec-22
Dec-23
Dec-24
Government bonds
Domestic equity
Foreign assets

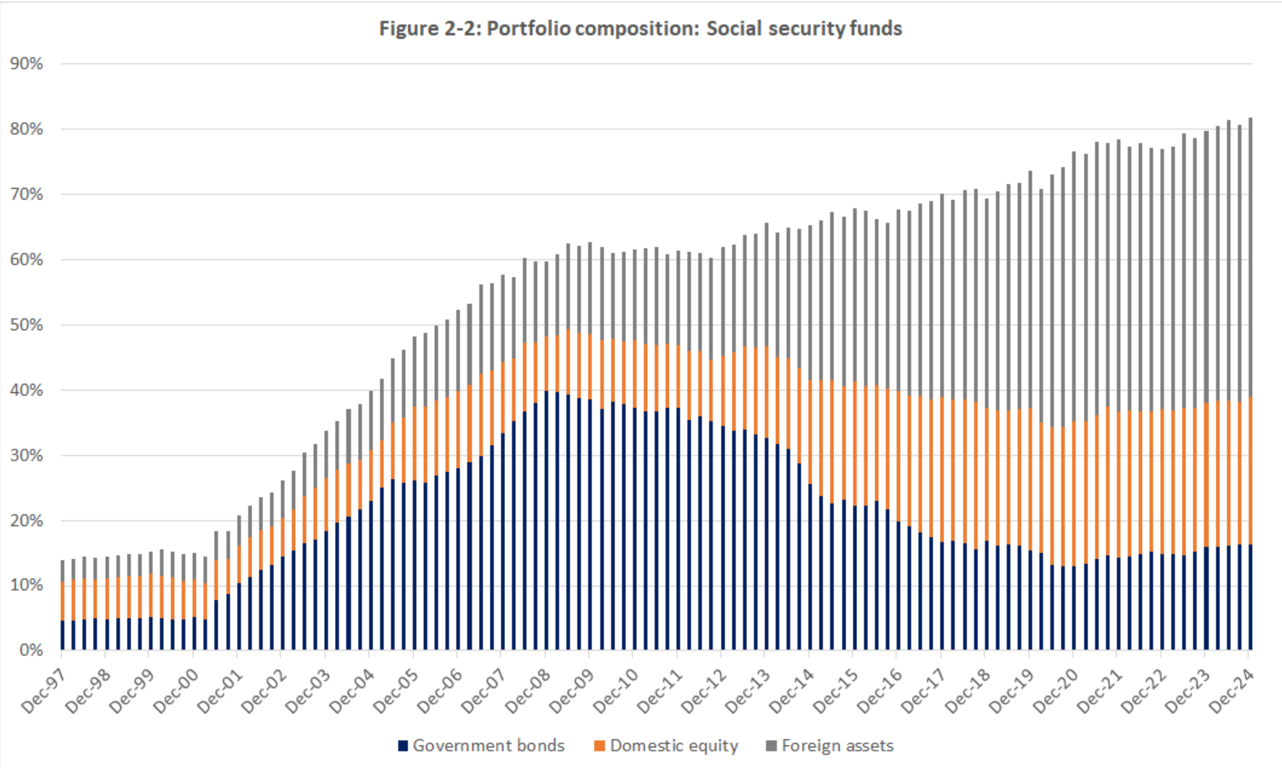


Figure 2-2: Portfolio composition: Social security funds
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Dec-97
Dec-98
Dec-99
Dec-00
Dec-01
Dec-02
Dec-03
Dec-04
Dec-05
Dec-06
Dec-07
Dec-08
Dec-09
Dec-10
Dec-11
Dec-12
Dec-13
Dec-14
Dec-15
Dec-16
Dec-17
Dec-18
Dec-19
Dec-20
Dec-21
Dec-22
Dec-23
Dec-24
Government bonds
Domestic equity
Foreign assets

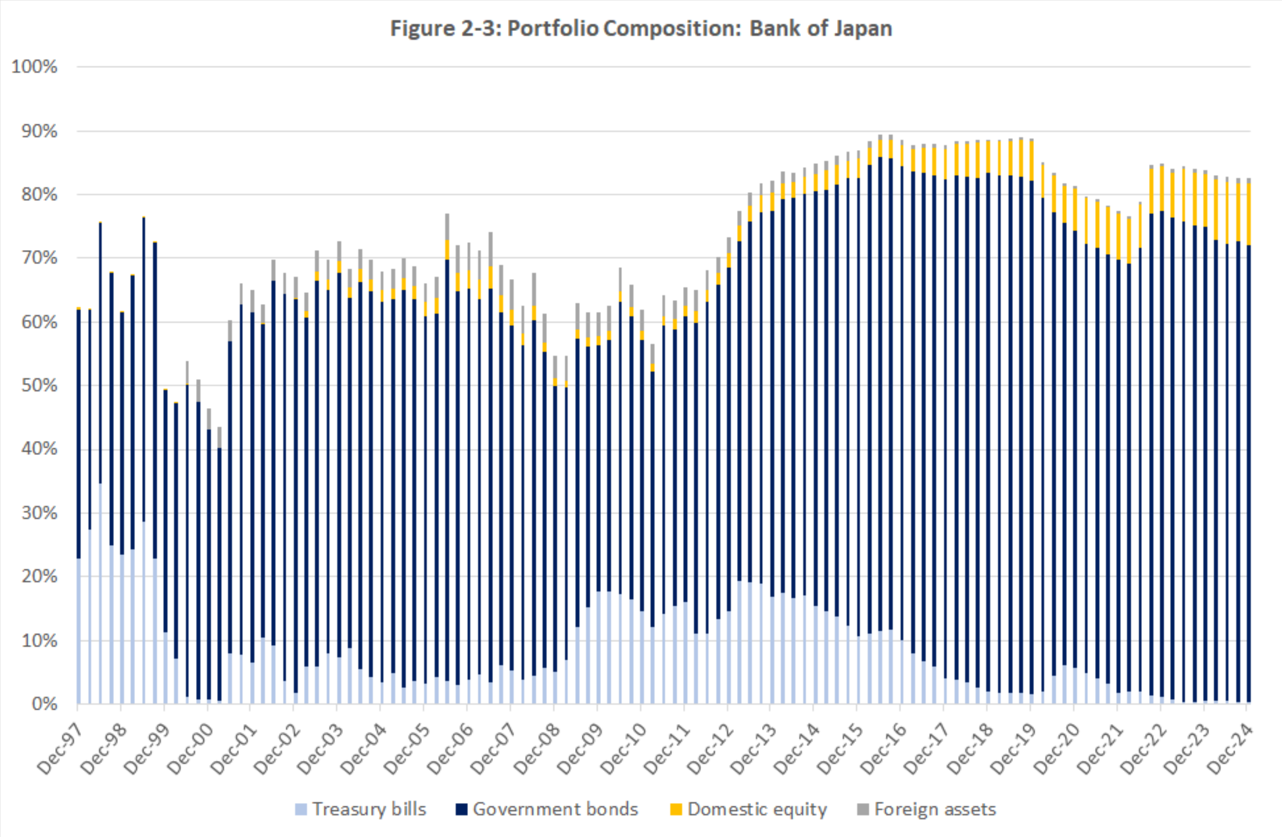


Figure 2-3: Portfolio Composition: Bank of Japan
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Dec-97
Dec-98
Dec-99
Dec-00
Dec-01
Dec-02
Dec-03
Dec-04
Dec-05
Dec-06
Dec-07
Dec-08
Dec-09
Dec-10
Dec-11
Dec-12
Dec-13
Dec-14
Dec-15
Dec-16
Dec-17
Dec-18
Dec-19
Dec-20
Dec-21
Dec-22
Dec-23
Dec-24
Treasury bills
Government bonds
Domestic equity
Foreign assets

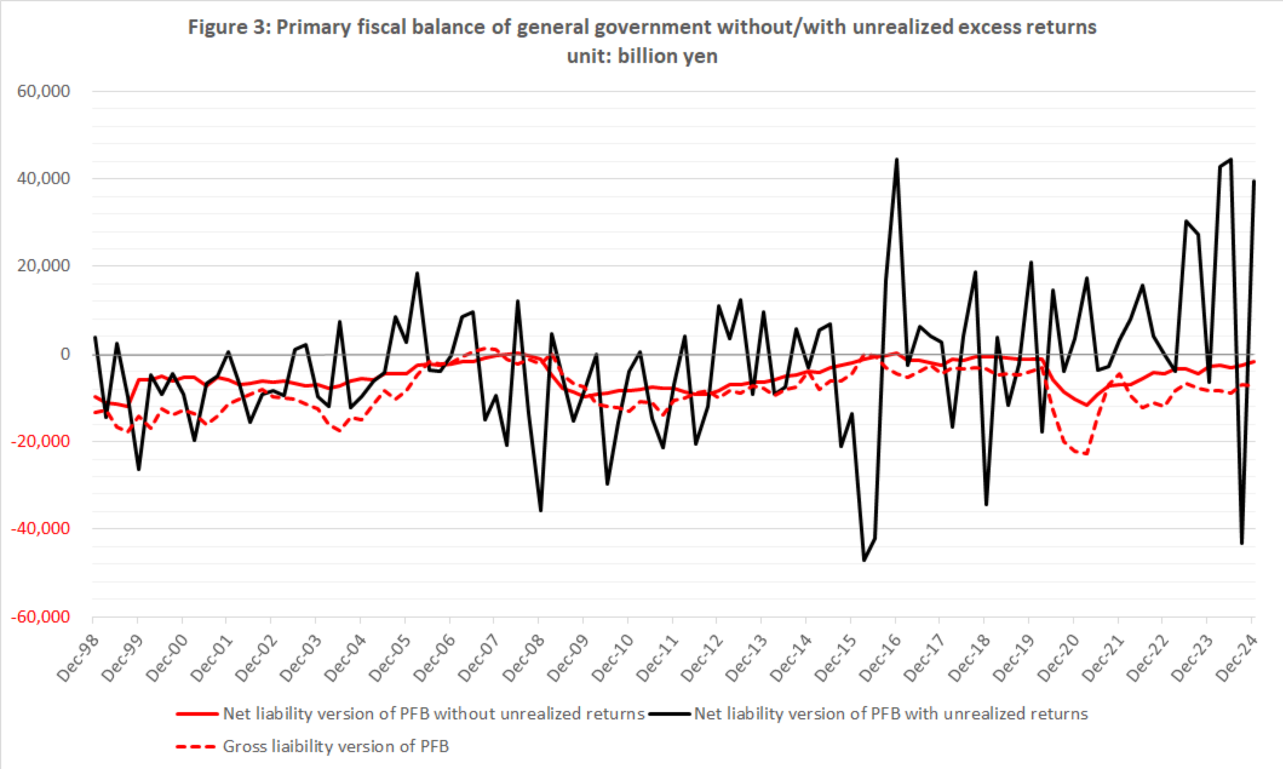


Figure 3: Primary fiscal balance of general government without/with unrealized excess returns
unit: billion yen
60,000
40,000
20,000
0
-20,000
-40,000
-60,000
Dec-98
Dec-99
Dec-00
Dec-01
Dec-02
Dec-03
Dec-04
Dec-05
Dec-06
Dec-07
Dec-08
Dec-09
Dec-10
Dec-11
Dec-12
Dec-13
Dec-14
Dec-15
Dec-16
Dec-17
Dec-18
Dec-19
Dec-20
Dec-21
Dec-22
Dec-23
Dec-24
Net liability version of PFB without unrealized returns
Net liability version of PFB with unrealized returns
Gross liaibility version of PFB

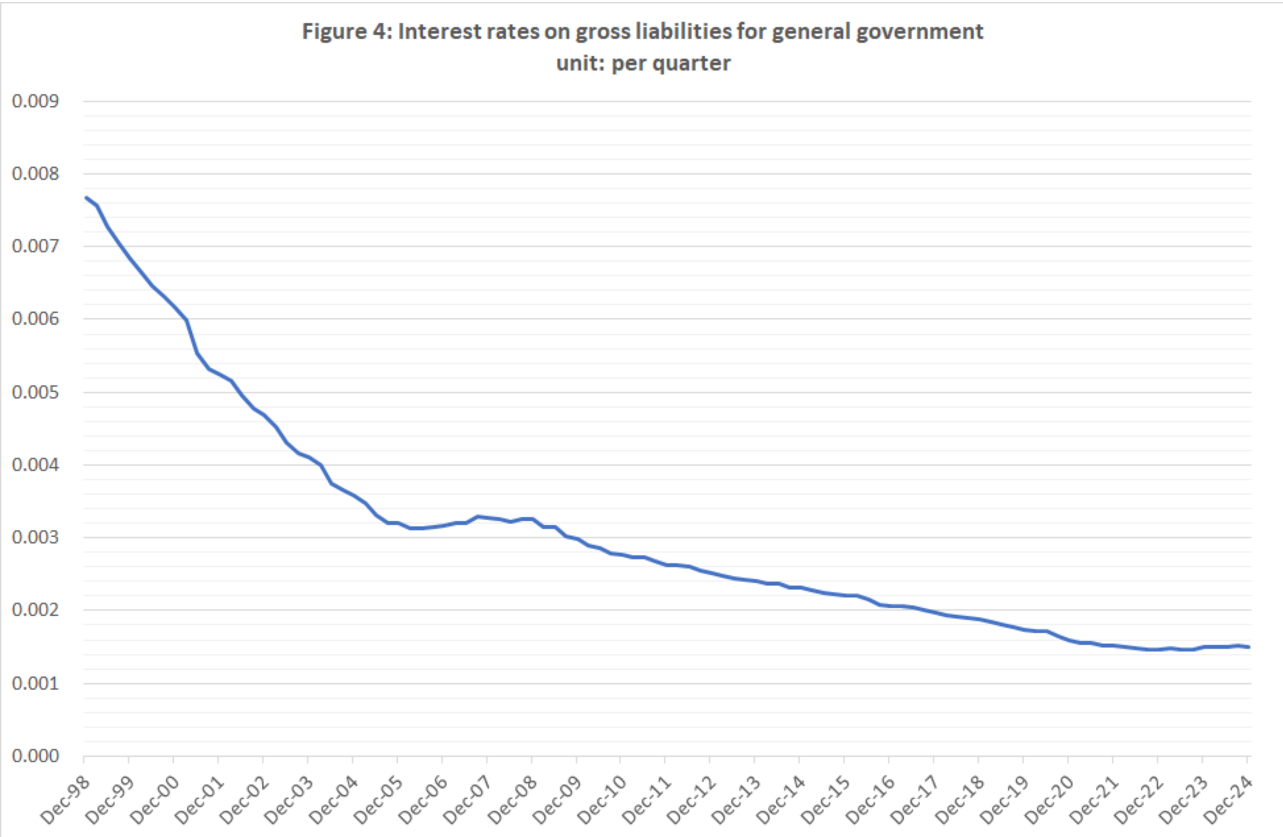


Figure 4: Interest rates on gross liabilities for general government
unit: per quarter
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0.000
Dec-98
Dec-99
Dec-00
Dec-01
Dec-02
Dec-03
Dec-04
Dec-05
Dec-06
Dec-07
Dec-08
Dec-09
Dec-10
Dec-11
Dec-12
Dec-13
Dec-14
Dec-15
Dec-16
Dec-17
Dec-18
Dec-19
Dec-20
Dec-21
Dec-22
Dec-23
Dec-24

**Figure 5-1: Realized primary fiscal balance on general government's gross liabilities against excess market returns (1999-2022)**

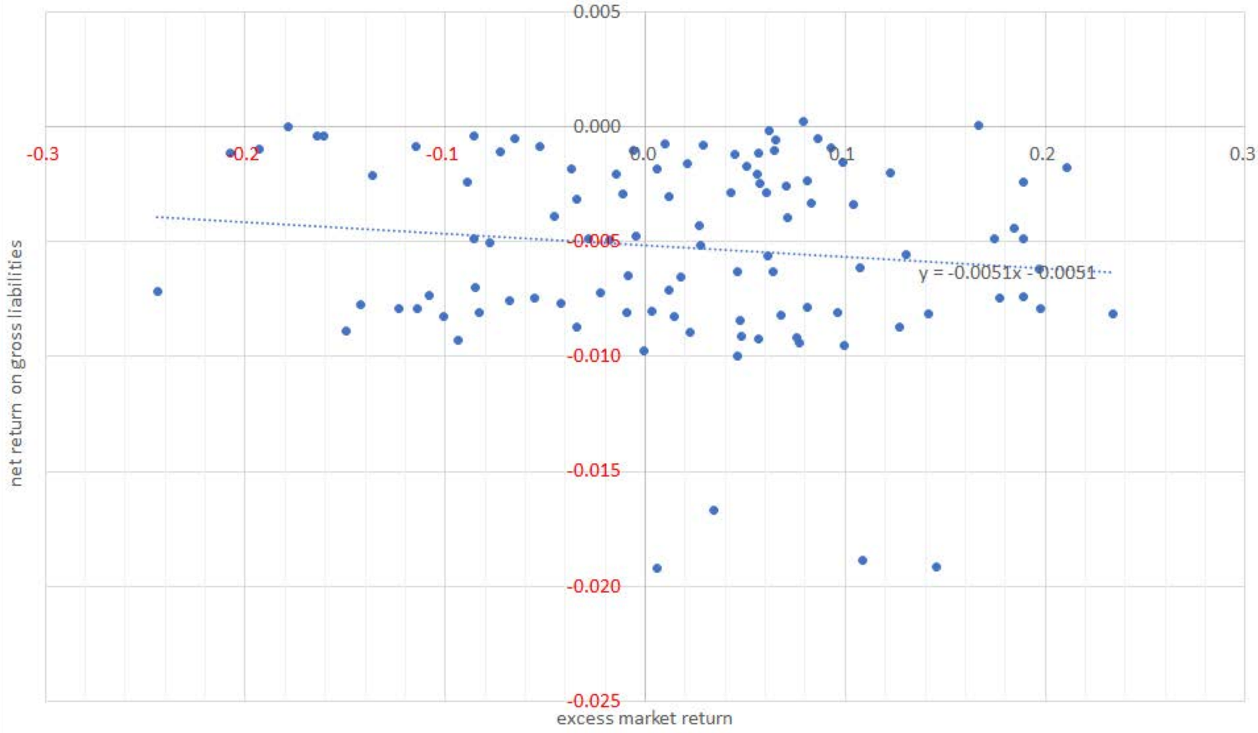
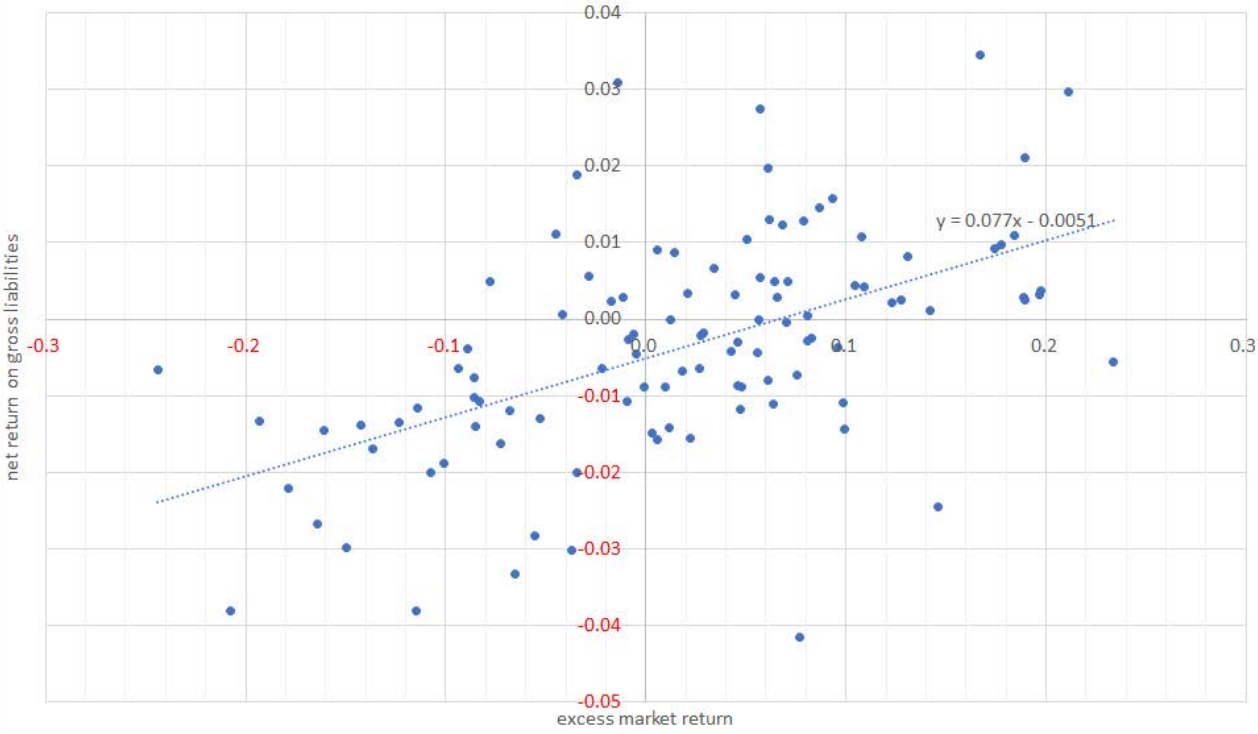



**Figure 5-2: Realized and unrealized primary fiscal balance on general government's gross liabilities against excess market returns (1999-2024)**



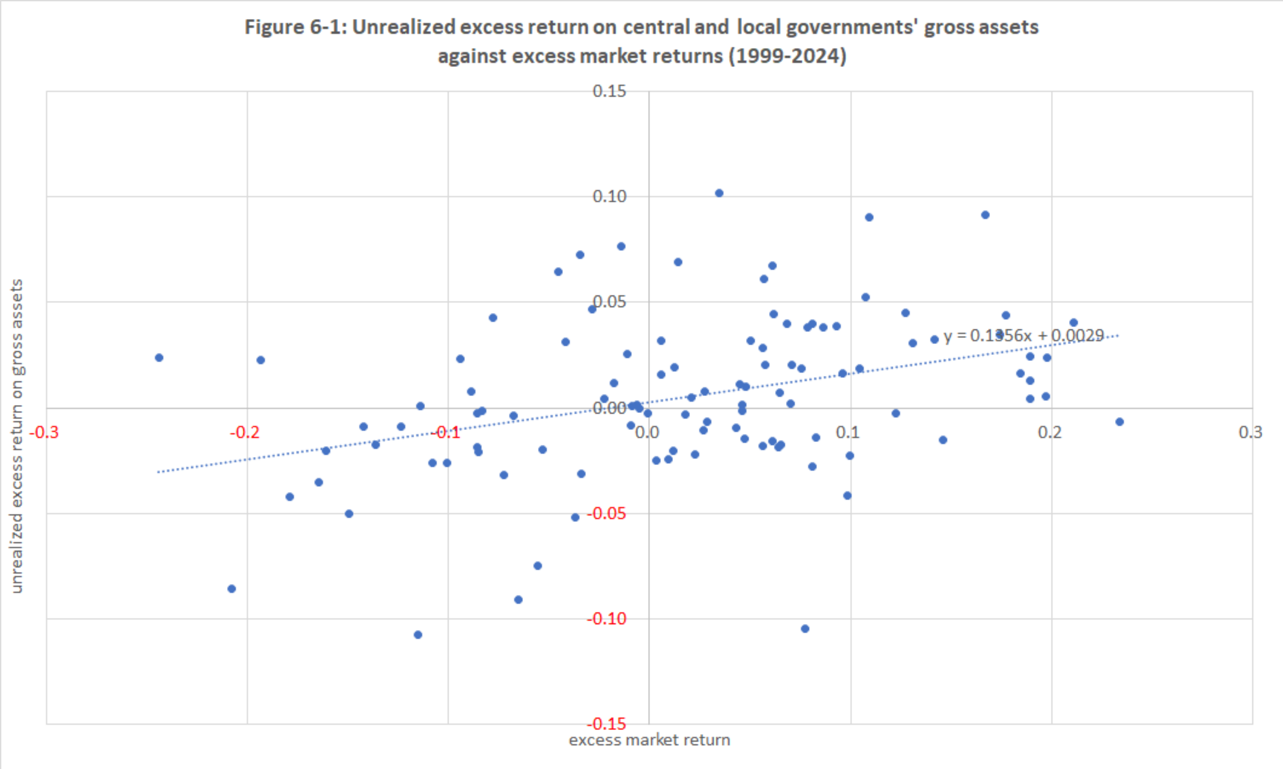


Figure 6-1: Unrealized excess return on central and local governments' gross assets against excess market returns (1999-2024)
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
y = 0.1356x + 0.0029
unrealized excess return on gross assets
excess market return

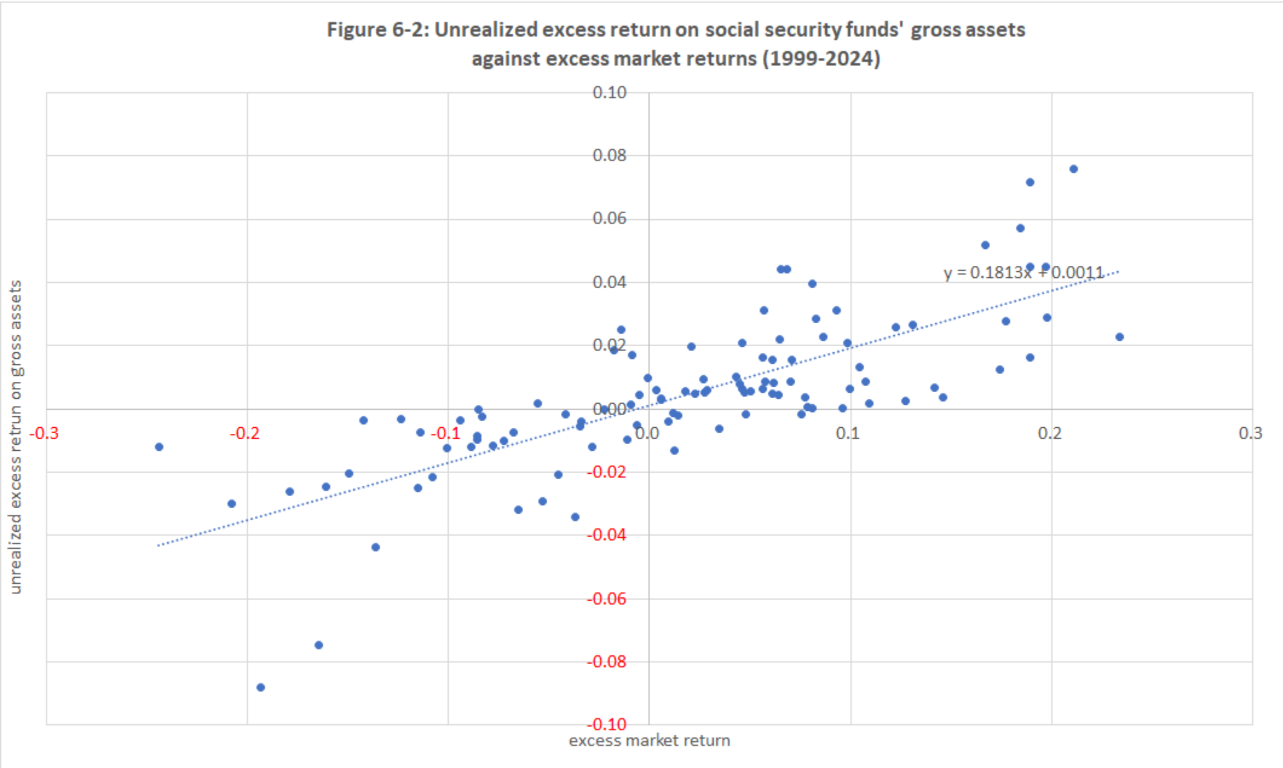


Figure 6-2: Unrealized excess return on social security funds' gross assets against excess market returns (1999-2024)
0.10
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
-0.10
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
y = 0.1813x + 0.0011
unrealized excess retrun on gross assets
excess market return

**Figure 6-3: Realized and unrealized net return on BOJ's net assets against excess market returns (1999-2024)**

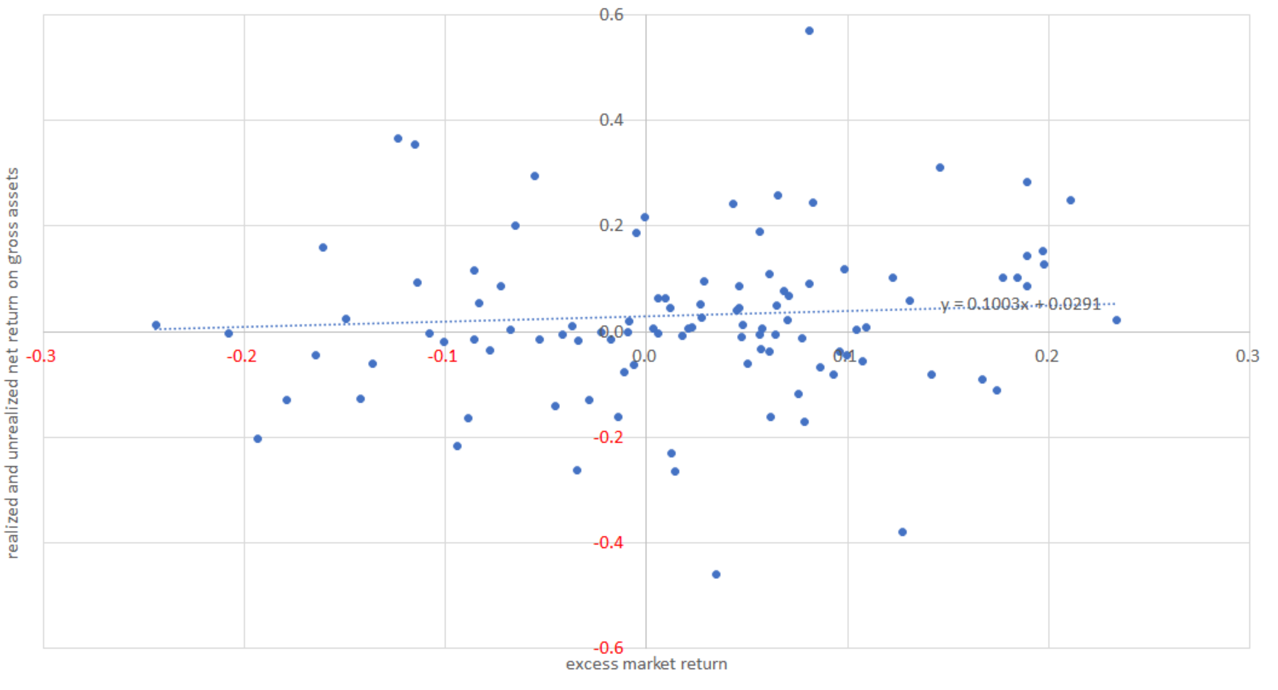
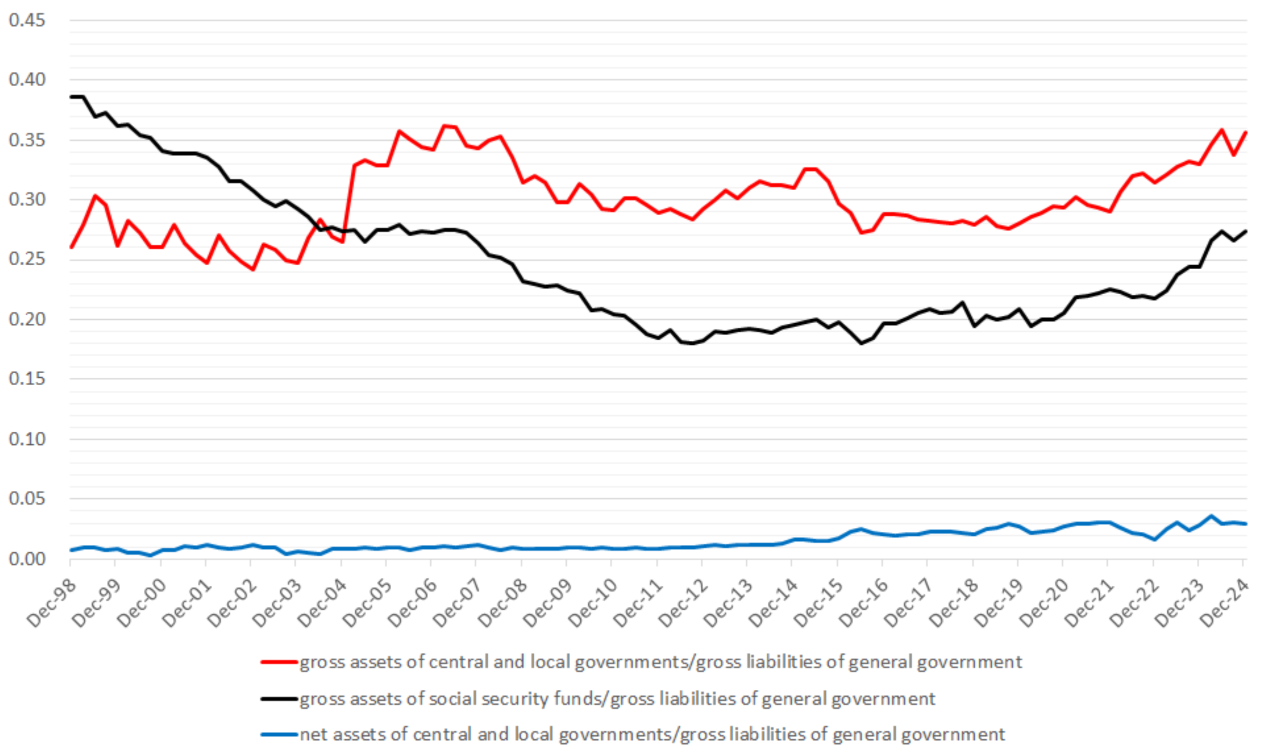



**Figure 7: The share of gross assets of CLGs and SSFs, and net assets of BOJ relative to gross liabilities of GG**



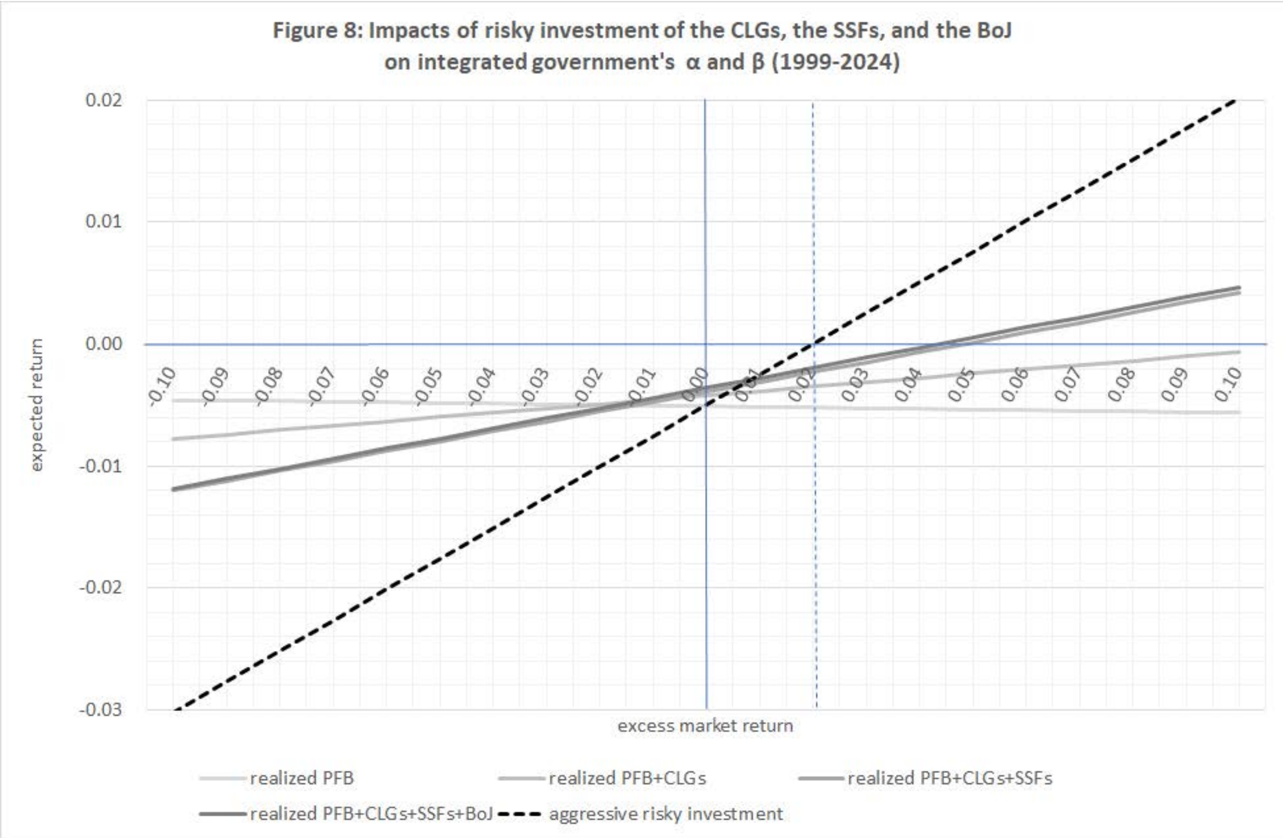


Figure 8: Impacts of risky investment of the CLGs, the SSFs, and the BoJ
on integrated government's α and β (1999-2024)
0.02
0.01
0.00
-0.01
-0.02
-0.03
expected return
-0.10
-0.09
-0.08
-0.07
-0.06
-0.05
-0.04
-0.03
-0.02
-0.01
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
excess market return
realized PFB
realized PFB+CLGs
realized PFB+CLGs+SSFs
realized PFB+CLGs+SSFs+BoJ
aggressive risky investment

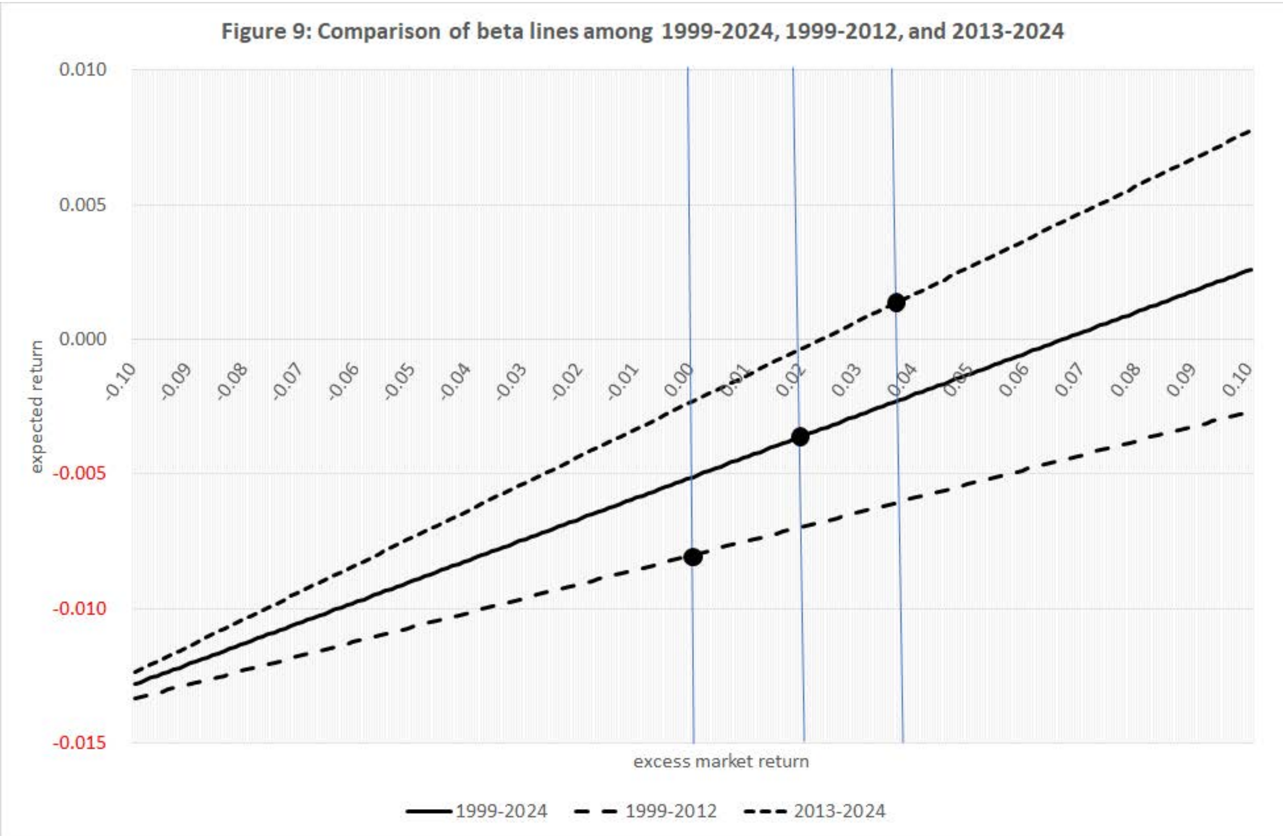


Figure 9: Comparison of beta lines among 1999-2024, 1999-2012, and 2013-2024
0.010
0.005
0.000
-0.005
-0.010
-0.015
expected return
-0.10
-0.09
-0.08
-0.07
-0.06
-0.05
-0.04
-0.03
-0.02
-0.01
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
excess market return
1999-2024
1999-2012
2013-2024